Code MIGALE state-of-the-art

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HiOCFD4
4th International Workshop on High-Order CFD Method
Foundation for Research and Technology Hellas (FORTH), Heraklion (Crete)
4th June 2016
...with the contribution of

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Towards Industrial LES/DNS in Aeronautics
Paving the Way for Future Accurate CFD
grant agreement No.635962
Brief code summary

- Discontinuous Galerkin (DG) method on hybrid grids
- Physical frame orthonormal basis functions
- 2D/3D steady and unsteady compressible and incompressible flows
- Explicit and implicit time accurate integration
- Fixed or rotating frame of reference
- Euler
- Navier–Stokes
- RANS coupled with the k-ω (EARSM)
- Hybrid RANS/LES (X-LES)
- MPI parallelism
- Fortran language
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**Implicit accurate time integration**

Several high-order temporal schemes are implemented

- Modified Extended BDF
- Two Implicit Advanced Step-point (TIAS)
- Explicit Singly Diagonally Implicit R-K (ESDIRK)
- linearly implicit Rosenbrock method

\[ \text{non-linear systems solution} \]

\[ \text{linear systems solution (here via GMRES)} \]

\[ i) \text{ Hi-O schemes are more efficient than Lo-O ones for high required accuracy} \]

\[ ii) \text{ Rosenbrock-type schemes are appealing both for accuracy and efficiency} \]

Convection of an isentropic vortex P6 solution on 50X50 el.
Rosenbrock schemes in a nutshell (I/II)

From the DG spatial discretization we obtain a system of non-linear ODEs or DAEs

\[ \mathbf{M}_P(\mathbf{W}) \frac{d\mathbf{W}}{dt} + \mathbf{R}(\mathbf{W}) = 0 \]

\[ \tilde{\mathbf{R}} = \mathbf{M}_P^{-1}\mathbf{R} \]

\[ \mathbf{W}^{n+1} = \mathbf{W}^n + \sum_{j=1}^{s} m_j \mathbf{Y}_j \]

\[ \left( \frac{\mathbf{M}_P}{\gamma \Delta t} + \mathbf{J} - \frac{\partial \mathbf{M}_P}{\partial \mathbf{W}} \tilde{\mathbf{R}} \right)^n \mathbf{Y}_i = -\mathbf{M}_P^n \left[ \tilde{\mathbf{R}} \left( \mathbf{W}^n + \sum_{j=1}^{i-1} a_{ij} \mathbf{Y}_j \right) - \sum_{j=1}^{i-1} \frac{c_{ij}}{\Delta t} \mathbf{Y}_j \right] \]

\[ i = 1, \ldots, s \]

only a linear system need to be solved for each stage

i.e. the Jacobian \( \mathbf{J} = \partial \mathbf{R}/\partial \mathbf{W} \) is assembled and factored only once per time step

With orthonormal basis functions (physical space) \( \mathbf{M}_P \) reduces to the identity for compressible flows with conservative variables

For other sets of variables their DOFs can be coupled within \( \mathbf{M}_P \) thus resulting in a matrix which can not be diagonal
Several Rosenbrock schemes, from order two to order six, have been compared.

No need to “exactly” solve systems: GMRES tolerance can be increased with confidence with a significant reduction of WU.

For a given order of accuracy, among the schemes considered, those with more stages are more accurate and efficient, e.g. RO5-8 vs. RO6-6.

Convection of an isentropic vortex P^6 solution on 50X50 el.

**Working variables**

Primitive variables to ensure the positivity of all thermodynamic variables at the discrete level.

We work with polynomial approximations not directly for \( p \) and \( T \) but for their logarithms \( \widetilde{p} = \log(p) \) and \( \widetilde{T} = \log(T) \)

In this way the computed values \( p = e^{\widetilde{p}} \) and \( T = e^{\widetilde{T}} \) are always positive

Easy to implement: almost only change \( \mathbf{M}_{P} \) ...
eXtra-Large Eddy Simulation (X-LES) in a nutshell (I/II)

Pros

• hybrid RANS\LES formulation independent from the wall distance
• use in LES mode of a clearly defined SGS based on the k-equation
• use of a k-ω turbulence model integrated to the wall

Cons

the filter width parameter is often related to the local element size

\[
\begin{align*}
\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho u_j k) &= \frac{\partial}{\partial x_j} \left[ (\mu + \sigma \bar{\mu}_t) \frac{\partial k}{\partial x_j} \right] + P_k - D_k \\
\frac{\partial}{\partial t}(\rho \bar{\omega}) + \frac{\partial}{\partial x_j}(\rho u_j \bar{\omega}) &= \frac{\partial}{\partial x_j} \left[ (\mu + \sigma \bar{\mu}_t) \frac{\partial \bar{\omega}}{\partial x_j} \right] + (\mu + \sigma \bar{\mu}_t) \left( \frac{\partial \bar{\omega}}{\partial x_k} \frac{\partial \bar{\omega}}{\partial x_k} \right) \\
&\quad + P_\omega - D_\omega + C_D 
\end{align*}
\]

...an “original” interpretation for the X-LES implementation...

**eXtra-Large Eddy Simulation (X-LES) in a nutshell (II/II)**

\[
\frac{\partial}{\partial t} (\rho k) + \frac{\partial}{\partial x_j} (\rho u_j k) = \frac{\partial}{\partial x_j} \left[ (\mu + \sigma^* \bar{\mu}_t) \frac{\partial k}{\partial x_j} \right] + P_k - D_k
\]

\[
\bar{\mu}_t = \alpha^* \frac{\rho \bar{k}}{\bar{\omega}} \quad D_k = \beta^* \rho \bar{k} \bar{\omega} \quad \bar{k} = \max(0, k)
\]

\[
\hat{\omega} = \max \left( e^{\tilde{\omega}_r}, \frac{\sqrt{\bar{k}}}{C_1 \Delta} \right)
\]

<table>
<thead>
<tr>
<th></th>
<th>RANS</th>
<th>LES</th>
<th>ILES</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{\mu}_t )</td>
<td>( \alpha^* \frac{\rho \bar{k}}{\bar{\omega}} )</td>
<td>( \alpha^* \rho \sqrt{\bar{k}} C_1 \Delta )</td>
<td>0</td>
</tr>
<tr>
<td>( D_k )</td>
<td>( \beta^* \rho k e^{\tilde{\omega}_r} )</td>
<td>( \beta^* \rho \frac{\bar{k}^{3/2}}{C_1 \Delta} )</td>
<td>0</td>
</tr>
</tbody>
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**X-LES of a shock BL interaction on a swept bump (AR2)**

$P^2$ converged computations with RANS+$k$-$\omega$ (also in its low-Re version) and EARSM1 have been performed and used as initialization for X-LES

- **Inlet boundary conditions**
  - $p_{0i} = 92000$ Pa
  - $T_{0i} = 300$ K
  - $Re_H = 1.69 \times 10^6$

- **Outlet static pressure** used to impose the shock position (*model dependent*)

- LBE to quickly find the “right” pressure ratio

- RO3-3 for the time-accurate solution

- Filter width $\Delta = 5e-2$ (*strong influence*)

- 96 cores of our in-house Intel cluster :-(

72960 hexahedral elements
The profiles are in reasonable agreement with the experiments and the numerical results of Cahen et al.
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X-LES of the transonic flow field in the NASA Rotor 37

- $P^2$ computation using RO3-3
- Filter width $\Delta = 5e^{-5}$
- Boundary conditions
  - $p_{01} = 101325$ Pa
  - $T_{01} = 288$ K
  - $\omega = 1800$ rad/s
  - $Tu_1 = 3\%$
  - $\alpha_1 = 0^\circ$
- still 96 cores of our in-house Intel cluster :-(

According to our first experiences, for a practical usage of X-LES, initializing with RANS seems mandatory
X-LES of the transonic flow field in the NASA Rotor 37

RANS  

X-LES ave.  

X-LES inst.  

30% span  

30% span  

30% span  

30% span  

30% span  

30% span
X-LES of the transonic flow field in the NASA Rotor 37
X-LES of the transonic flow field in the NASA Rotor 37

Spanwise distributions

Initializing the solution from a flow field corresponding to a normalized mass flow, resulting from a RANS computation, of ≈0.98
X-LES moved towards ≈0.996
Thursday, June 9 8:30-10:30 - MS 905 - 2 (Room 20)
Alessandra Nigro, Carmine De Bartolo, Andrea Crivellini, Francesco Bassi
MATRIX-FREE MODIFIED EXTENDED BACKWARD DIFFERENTIATION FORMULAE
APPLIED TO THE DISCONTINUOUS GALERKIN SOLUTION OF COMPRESSIBLE UNSTEADY VISCIOUS FLOWS

Thursday, June 9 14:30-16:30 - MS 905 - 3 (Room 20)
Francesco Carlo Massa, Gianmaria Noventa, Francesco Bassi, Alessandro Colombo,
Antonio Ghidoni, Marco Lorini
HIGH-ORDER LINEARLY IMPLICIT TWO-STEP PEER METHODS FOR
THE DISCONTINUOUS GALERKIN SOLUTION OF THE INCOMPRESSIBLE RANS EQUATIONS

Thursday, June 9 17:00-19:00 - MS 905 - 4 (Room 20)
Antonio Ghidoni, Marco Lorini, Gianmaria Noventa, Francesco Bassi, Alessandro Colombo
DISCONTINUOUS GALERKIN SOLUTION OF THE REYNOLDS- AVERAGED NAVIER–STOKES AND
KL-KT-LOG(W) TRANSITION MODEL EQUATIONS

Friday, June 10 9:00-11:00 - MS 910 - 2 (Room 15)
Francesco Bassi, Lorenzo Botti, Alessandro Colombo, Andrea Crivellini, Antonio Ghidoni,
Marco Lorini, Francesco Carlo Massa, Gianmaria Noventa
ON THE IMPLEMENTATION OF X-LES IN A HIGH-ORDER IMPLICIT DG SOLVER

Tuesday, June 7 8:30-10:30 - CS 930 - 3 (Room 15)
Francesco Bassi, Alessandro Colombo, Andrea Crivellini, Matteo Franciolini
HYBRID OPENMP/MPI PARALLELIZATION OF A HIGH–ORDER DISCONTINUOUS GALERKIN CFD SOLVER