DNS of the Taylor-Green vortex at Re=1600

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Introduction

This problem is aimed at testing the accuracy and the performance of high-order methods on the direct numerical simulation of a three-dimensional periodic and transitional flow defined by a simple initial condition: the Taylor-Green vortex. The initial flow field is given by

$$\begin{aligned} u &= V_0 \sin\left(\frac{x}{L}\right) \cos\left(\frac{y}{L}\right) \cos\left(\frac{z}{L}\right) \ , \\ v &= -V_0 \cos\left(\frac{x}{L}\right) \sin\left(\frac{y}{L}\right) \cos\left(\frac{z}{L}\right) \ , \\ w &= 0 \ , \\ p &= p_0 + \frac{\rho_0 V_0^2}{16} \left(\cos\left(\frac{2x}{L}\right) + \cos\left(\frac{2y}{L}\right)\right) \left(\cos\left(\frac{2z}{L}\right) + 2\right) \ . \end{aligned}$$

This flow transitions to turbulence, with the creation of small scales, followed by a decay phase similar to decaying homogeneous turbulence (yet here non isotropic), see figure 1.

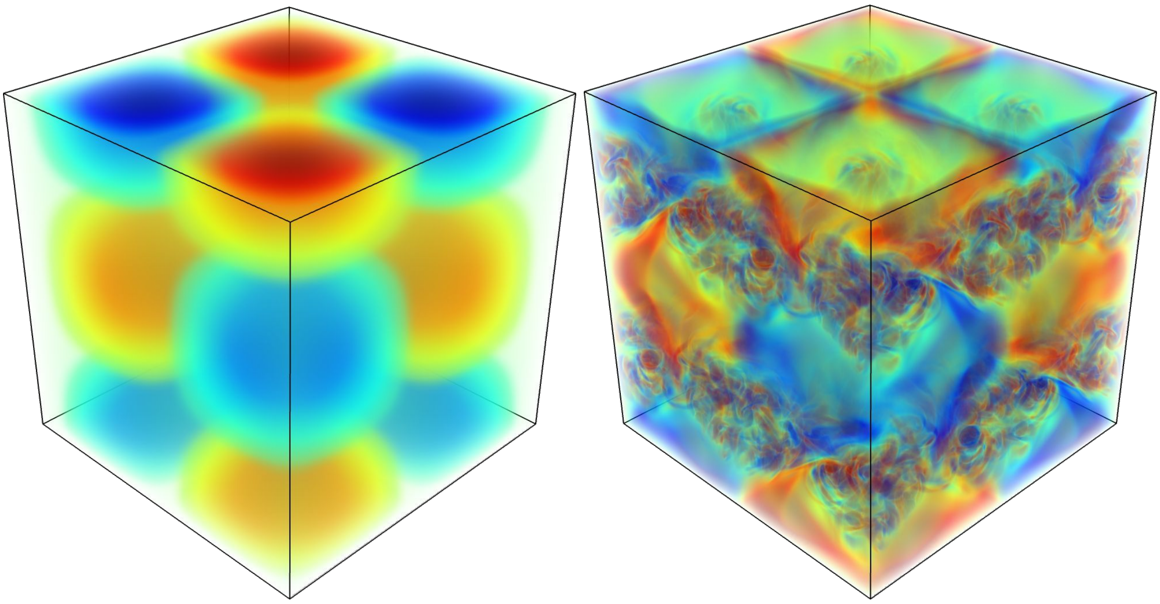

Figure 1. Volume rendering of the z-vorticity at t=0 and t_{final}=10.

For compressible codes, the velocity V_0 should be fixed such that the Mach number based on this velocity does not exceed 0.1.

Required simulations/data

Two computational campaigns are proposed: one (mandatory) on structured grids and one (optional) on unstructured grids. The participants should provide the following statistics:

- Monitors:
  - Temporal evolution of the kinetic energy integrated on the domain:

$$E_k = \frac{1}{\rho_0 \, \Omega} \int_\Omega \rho \, \frac{\boldsymbol{v} \cdot \boldsymbol{v}}{2} \, d\Omega$$

  - Temporal evolution of the kinetic energy dissipation rate:

$$\epsilon = -\frac{dE_k}{dt}$$

  - Temporal evolution of the dissipation rate based on the enstrophy integrated on the domain:

$$\mathcal{E} = \frac{1}{\rho_0 \, \Omega} \int_\Omega \rho \, \frac{\boldsymbol{\omega} \cdot \boldsymbol{\omega}}{2} \, d\Omega \qquad \epsilon = 2 \frac{\mu}{\rho_0} \, \mathcal{E}$$

- The vorticity norm on the periodic face x/L= −π at time t/tc=8. An illustration is given in figure 2.

Figure 2. Iso-contours of the dimensionless vorticity norm. Comparison between the results obtained with PSM and DGM(4).

- Kinetic energy spectra time at t/tc=8 (Figure 3).

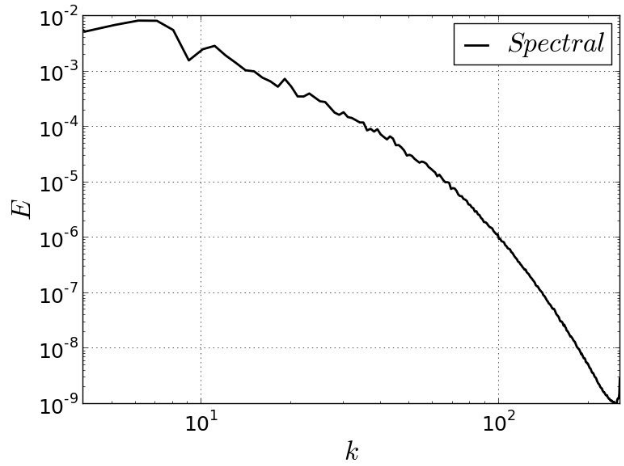

Figure 3. Energy spectra at t/tc=8

Detailed description

Governing Equations and flow conditions

The Reynolds number of the flow is here defined as Re = $\rho_0 V_0 L/\ \mu$ and is equal to 1600.

Compressible or incompressible Navier-Stokes equations can be used for this benchmark. For incompressible solvers (i.e., ρ = ρ0), Navier-Stokes equations with constant physical properties can be used. Then, one also does not need to compute the temperature field as the temperature field plays no role in the fluid dynamics. For compressible solvers, the Mach number based on the V_0 velocity is set to M=0.1 to match incompressible conditions. The Prandtl number is Pr=0.71. The initial temperature field is taken uniform: $T = T_0$; thus, the initial density field is taken as $\rho = R\ T_0$.

The physical duration of the computation is based on the characteristic convective time tc = and is set to tfinal = 10 tc, as the maximum of the dissipation (and thus the smallest turbulent structures) occurs at t ≈ 8 tc.

Geometry, grids and expected results

The flow is computed within a periodic square box defined as $-\pi L \le x, y, z \le \pi L$. Grid convergence studies on Cartesian meshes with equivalent resolutions 64^3, 128^3, 256^3 will be accepted [1], as well as order convergence studies on constant resolution. Furthermore similar studies on perturbed or unstructured meshes - a mandatory set will be provided by the test case leader on request - are expected as well.

File formats

Data should be non-dimensionalized using initial density ϱ_0, dynamic viscosity μ, initial velocity V_0 and reference length L.

Provide a clearly and coherently named set of files (ie. a single base name, but differring extensions, per computation). Each of the data files should have a header, structured as follows

 # participant:
 # code name:
 # mesh resolution:
 # discretization:
 # order of convergence:
 # mesh file name:
 # density:
 # velocity:
 # dynamic viscosity:
 # reference length:

Energy evolution

The file should contain the following data

<time> <kinetic energy Ek> <dEk/dt> <ε>

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[1] In case of finite element methods, with polynomial interpolation of order p within each element, the respective resolutions should be $(64/p)^3$, $(128/p)^3$, $(256/p)^3$

The compressible contributions to the dissipation can be provided, but will not be discussed as they are negligeable for M0<=0.1. The time derivative should be computed by the scheme used for its discretisation, whereas the temporal resolution should correspond to that of the computation.

Energy spectrum

The spectrum should be computed up to the cut-off wave number corresponding to the equivalent resolution (eg. up to 128 for the 256 dof mesh).

The file should provide

<wave number> <spectral energy>