Test case BL3 - Heaving and pitching airfoil
4th International Workshop on High-Order CFD Methods

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UC Berkeley and U. Michigan

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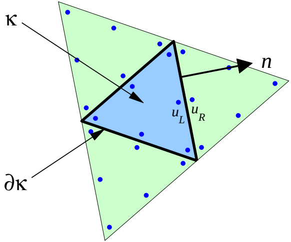
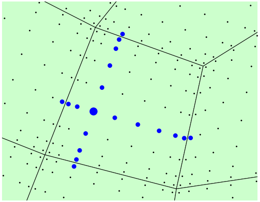
The 3DG Software Package

- Standard nodal high-order DG discretization for conservation laws
- Curved, fully unstructured meshes of tetrahedra/hexahedra
- General multiphysics formulation – same code used for fluid/structure, for DG/CG, etc, with modular interfaces
- Fully implicit formulation, sparse block formats for Jacobians
- Highly sparse "Line-DG" formulation for quad/hex elements





- Efficient parallel solvers with matrix-based decompositions

ALE Formulation for Deforming Domains

- Use mapping-based ALE formulation for moving domains [Visbal,Gaitonde 2002], [Persson,Bonet,Peraire 2009]
- Map from reference domain V to physical deformable domain $v(t)$
- Introduce the *mapping deformation gradient* $\boldsymbol{G} = \nabla_X \boldsymbol{\mathcal{G}}$ and the *mapping velocity* $\boldsymbol{v}_X = \frac{\partial \boldsymbol{\mathcal{G}}}{\partial t}\Big|_X$, and set $g = \det(\boldsymbol{G})$
- For numerically computed grid motions, compute *stage consistent* velocities by imposing

$$\boldsymbol{x}_i = \boldsymbol{x}_0 + \Delta t \sum_{j=1}^{s} a_{ij} \boldsymbol{\nu}_j \quad \Longrightarrow \quad \boldsymbol{\nu}_i = \sum_{j=1}^{s} (A^{-1})_{ij} \frac{\boldsymbol{x}_j - \boldsymbol{x}_0}{\Delta t}, \quad i = 1, \ldots, s,$$

  where A is the implicit Runge-Kutta Butcher tableaux [Froehle & Persson, 2014]
- Transform equations to account for the motion

Transformed Equations

- The system of conservation laws in the physical domain $v(t)$

$$\left.\frac{\partial \boldsymbol{U}_x}{\partial t}\right|_x + \boldsymbol{\nabla}_x \cdot \boldsymbol{F}_x(\boldsymbol{U}_x, \boldsymbol{\nabla}_x \boldsymbol{U}_x) = 0$$

can be written in the reference configuration V as

$$\left.\frac{\partial \boldsymbol{U}_X}{\partial t}\right|_X + \boldsymbol{\nabla}_X \cdot \boldsymbol{F}_X(\boldsymbol{U}_X, \boldsymbol{\nabla}_X \boldsymbol{U}_X) = 0$$

where

$$\boldsymbol{U}_X = g\boldsymbol{U}_x \;, \qquad \boldsymbol{F}_X = g\boldsymbol{G}^{-1}\boldsymbol{F}_x - \boldsymbol{U}_X \boldsymbol{G}^{-1}\boldsymbol{v}_X$$

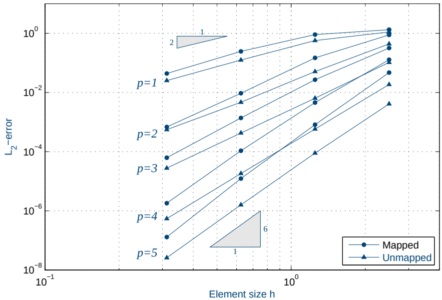
and

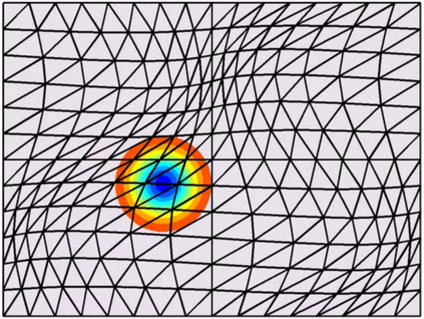
$$\boldsymbol{\nabla}_x \boldsymbol{U}_x = \boldsymbol{\nabla}_X (g^{-1}\boldsymbol{U}_X)\boldsymbol{G}^{-T} = (g^{-1}\boldsymbol{\nabla}_X \boldsymbol{U}_X - \boldsymbol{U}_X \boldsymbol{\nabla}_X (g^{-1}))\boldsymbol{G}^{-T}$$

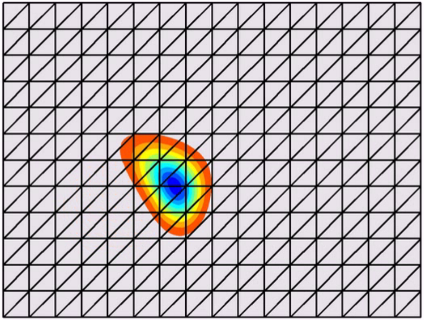
- Details in [Persson,Bonet,Peraire 2009], including how to satisfy the so-called Geometric Conservation Law (GCL)

ALE Formulation for Deforming Domains

- Mapping-based formulation gives arbitrarily high-order accuracy in space and time

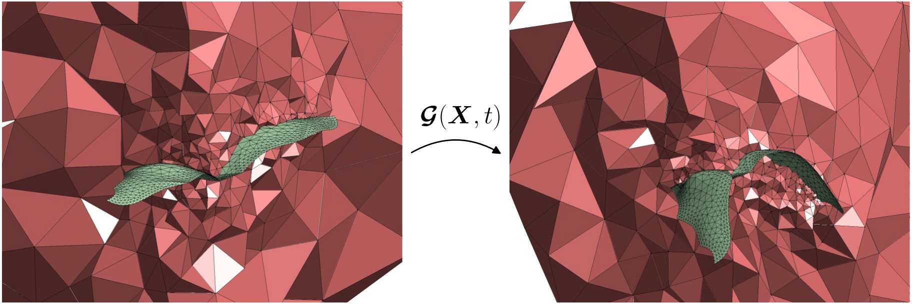




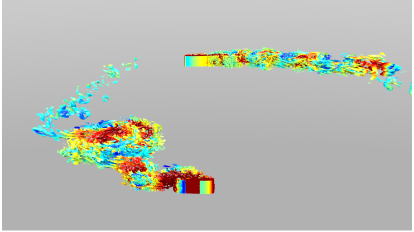


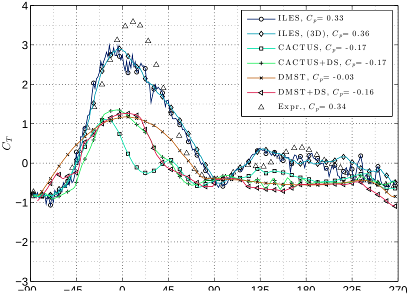
Nonlinear Elasticity for Deforming Domains

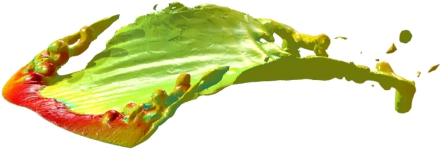
- *Non-linear solid mechanics approach*: [Persson & Peraire 2009]
  - An initial reference mesh corresponds to an undeformed solid
  - External forces come from the true moving boundary constraints
  - Solving for a force equilibrium gives the deformed (curved) boundary conforming mesh
- High-order ALE methods require a smooth mapping $\mathcal{G}(\boldsymbol{X}, t)$ such that the elements are aligned with the moving boundaries

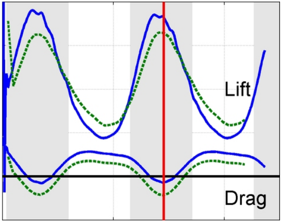


Moving Domain Applications



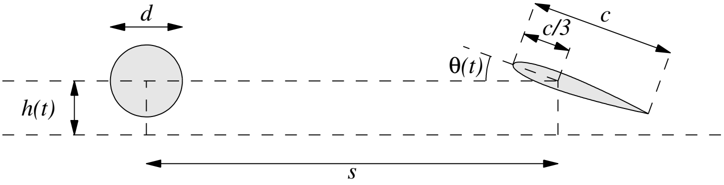






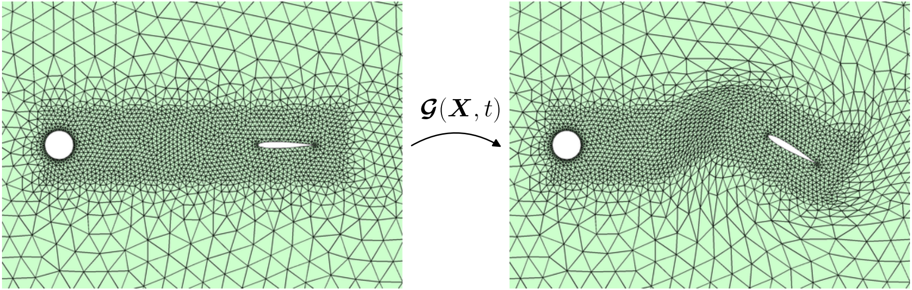
1st and 2nd High-Order Workshop

- Inspired by experimental study [Gopalkrishnan/Triantafyllou/et al, '94], computational study in [Persson/Peraire/Bonet '09]
- An oscillating cylinder produces vortices that interact with a heaving and pitching airfoil, in a typical flapping motion
- Freestream Mach = 0.2, Re = 500, St = 0.1 (for cylinder)
- Thrust on airfoil highly dependent on distance s and the vortices convected from cylinder – potentially good case for high-order methods



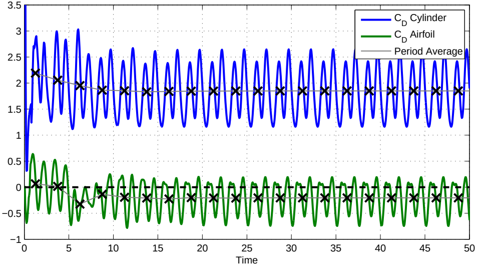
Mapping between reference and deforming domain

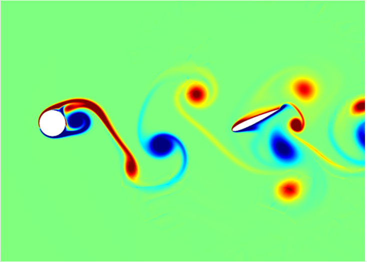
- Analytically prescribed mapping $\boldsymbol{\mathcal{G}}(\boldsymbol{X}, t)$
- Combination of rigid motions and smooth blending functions
- Symbolic computation of grid velocity $\boldsymbol{v}_X = \frac{\partial \boldsymbol{\mathcal{G}}}{\partial t}\Big|_X$ and deformation gradient $\boldsymbol{G} = \nabla_X \boldsymbol{\mathcal{G}}$
- No moving meshes $\longrightarrow$ high-order accuracy in space and time



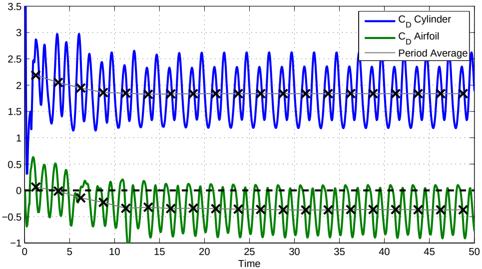
Results, St = 0.2 (drag coefficients)

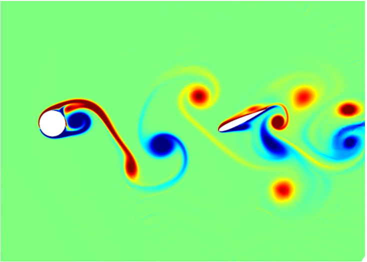
Airfoil position $s = 3.76$:





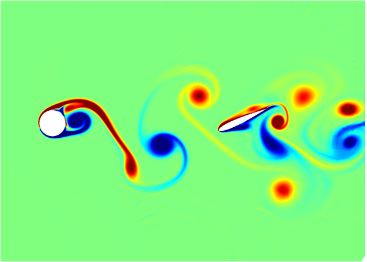
Airfoil position $s = 3.50$:





1st and 2nd High-Order Workshop, Summary

- Hard to compare the numbers due to complex flow behavior
- However, the results appear consistent
- The p-convergence by UCB and UMich indicate fully converged solutions
- Comparisons between polynomial degrees p indicate large benefits with high-order discretizations
- St = 0.2 promising for accurate quantitative comparisons
- Future improvements:
  - Use parameters that converge to time-periodic solutions
  - Possibly: Calculate *exactly* periodic solutions (hard)
  - More rigorous comparison of computational cost



Energetically Optimal Flapping, Thrust Constraint

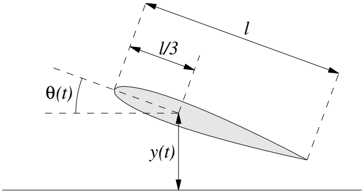
$$\underset{\boldsymbol{\mu}}{\text{minimize}} \quad -\int_{2T}^{3T}\int_{\boldsymbol{\Gamma}} \boldsymbol{f}\cdot\dot{\boldsymbol{x}}\,dS\,dt$$

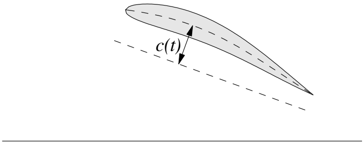
$$\text{subject to} \quad \int_{2T}^{3T}\int_{\boldsymbol{\Gamma}} \boldsymbol{f}\cdot\boldsymbol{e}_1\,dS\,dt = q$$

$$\boldsymbol{U}(\boldsymbol{x},0) = \bar{\boldsymbol{U}}(\boldsymbol{x})$$

$$\frac{\partial \boldsymbol{U}}{\partial t} + \nabla\cdot\boldsymbol{F}(\boldsymbol{U},\nabla\boldsymbol{U}) = 0$$

- Isentropic, compressible, Navier-Stokes
- Re = 1000, M = 0.2
- $y(t)$, $\theta(t)$, $c(t)$ parametrized via periodic cubic splines
- Black-box optimizer: SNOPT



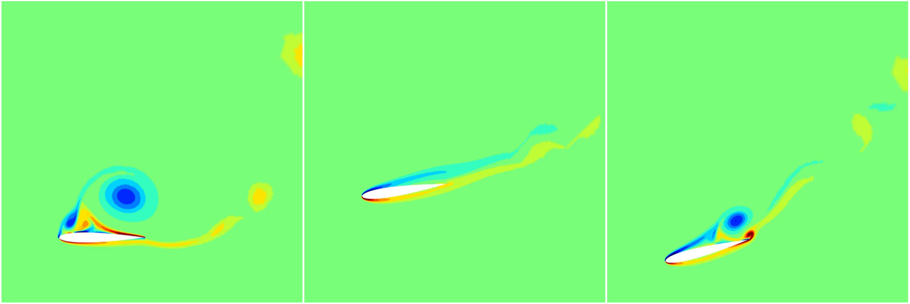


Airfoil schematic, kinematic description

Optimal Control - Fixed Shape

Fixed Shape, Optimal Rigid Body Motion (RBM), Varied x-Impulse

Initial Guess	Optimal RBM	Optimal RBM
Energy = 9.4096	Energy = 0.45695	Energy = 4.9475
x-impulse = -0.1766	x-impulse = 0.000	x-impulse = -2.500



Initial Guess

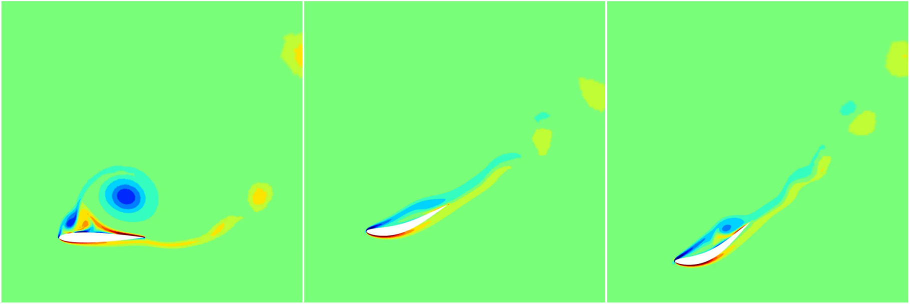
Optimal RBM $J_x = 0.0$

Optimal RBM $J_x = -2.5$

Optimal Control, Time-Morphed Geometry

Optimal Rigid Body Motion (RBM) and Time-Morphed Geometry (TMG), Varied x-Impulse

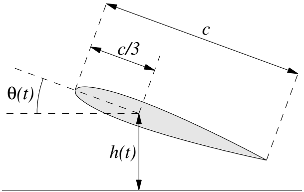
Initial Guess	Optimal RBM/TMG $J_x = 0.0$	Optimal RBM/TMG $J_x = -2.5$
Energy = 9.4096	Energy = 0.45027	Energy = 4.6182
x-impulse = -0.1766	x-impulse = 0.000	x-impulse = -2.500



Initial Guess — Optimal RBM/TMG $J_x = 0.0$ — Optimal RBM/TMG $J_x = -2.5$

HiOCFD4 BL3 - Heaving and pitching airfoil

- Deforming domain problem: NACA 0012 airfoil undergoing a flapping-type motion
- Freestream Mach = 0.2, Re = 1000
- Steady-state solution as initial condition
- Three different motions:



Case 1 (Pure heaving)

$$\begin{cases} h(t) = b_2(t) \\ \theta(t) = 0 \end{cases}$$

Case 2 (Flow aligning)

$$\begin{cases} h(t) = b_2(t) \\ \theta(t) = A_2 \cdot b_1(t) \end{cases}$$

Case 3 (Energy extracting)

$$\begin{cases} h(t) = b_3(t) \\ \theta(t) = A_3 \cdot b_1(t) \end{cases}$$

where $A_2 = 60\pi/180$, $A_3 = 80\pi/180$,
$b_1(t) = t^2(t^2 - 4t + 4)$, $b_2(t) = t^2(3 - t)/4$,
$b_3(t) = t^3(-8t^3 + 51t^2 - 111t + 84)/16$.

Output Quantities

- First output: The work (energy) which the fluid exerts on the airfoil during the motion:

$$W = \int_0^T \boldsymbol{F}(t) \cdot \boldsymbol{v}_0 \, dt + \int_0^T \boldsymbol{T}(t) \cdot \boldsymbol{\omega} \, dt = \int_0^T F_y(t) \dot{h}(t) \, dt + \int_0^T T_z(t) \dot{\theta}(t) \, dt.$$

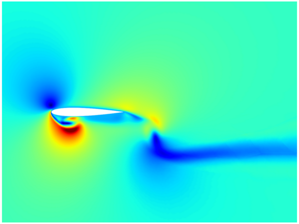
or

$$W = \int_0^T \int_{\text{airfoil}} \vec{v}_G(t) \cdot \vec{f}_{\text{surf}}(t) \, ds \, dt$$

- Second output: The vertical impulse from the fluid onto the airfoil during the motion:

$$I = \int_0^T F_y(t) \, dt$$

Solution Fields



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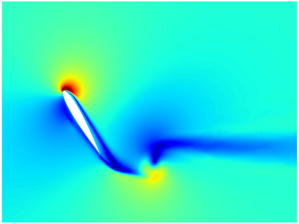
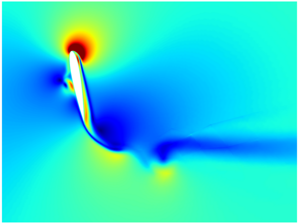


Case 2 (Flow aligning)

$$\begin{cases} h(t) = b_2(t) \\ \theta(t) = A_2 \cdot b_1(t) \end{cases}$$



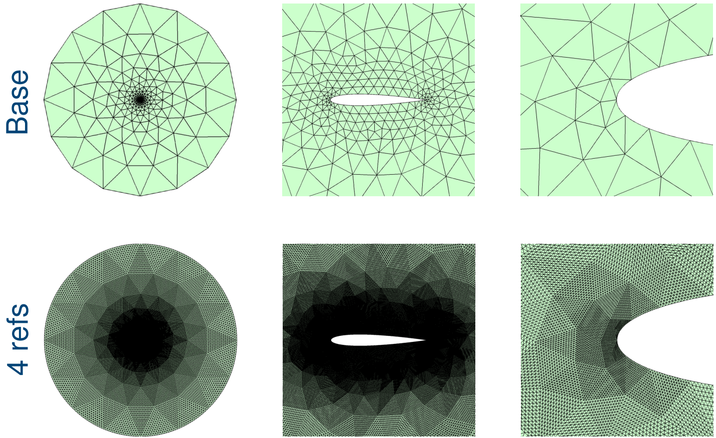
Case 3 (Energy extracting)

$$\begin{cases} h(t) = b_3(t) \\ \theta(t) = A_3 \cdot b_1(t) \end{cases}$$

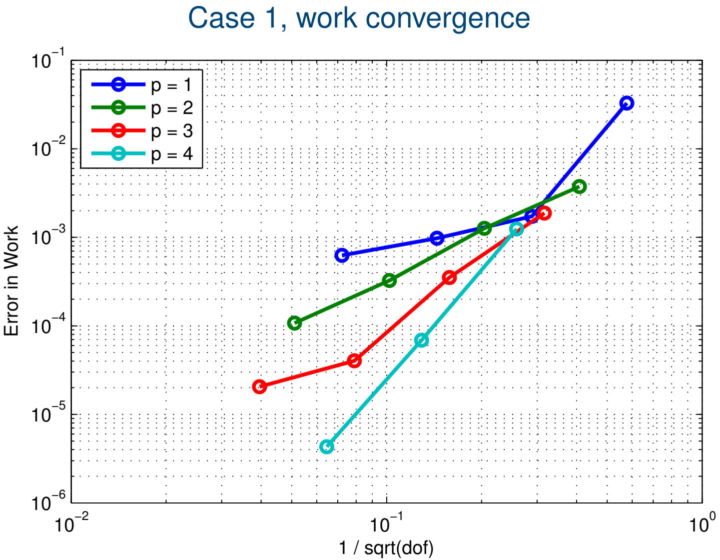
where $A_2 = 60\pi/180$, $A_3 = 80\pi/180$,
$b_1(t) = t^2(t^2 - 4t + 4)$, $b_2(t) = t^2(3 - t)/4$,
$b_3(t) = t^3(-8t^3 + 51t^2 - 111t + 84)/16$.

UC Berkeley Results – Meshes

- Base mesh: 971 triangular elements
- Up to four uniform refinements, polynomial degrees $p = 1, 2, 3, 4$
- 3rd order DIRK scheme in time, $\Delta t = 2 \cdot 10^{-3}$



UC Berkeley Results – Convergence



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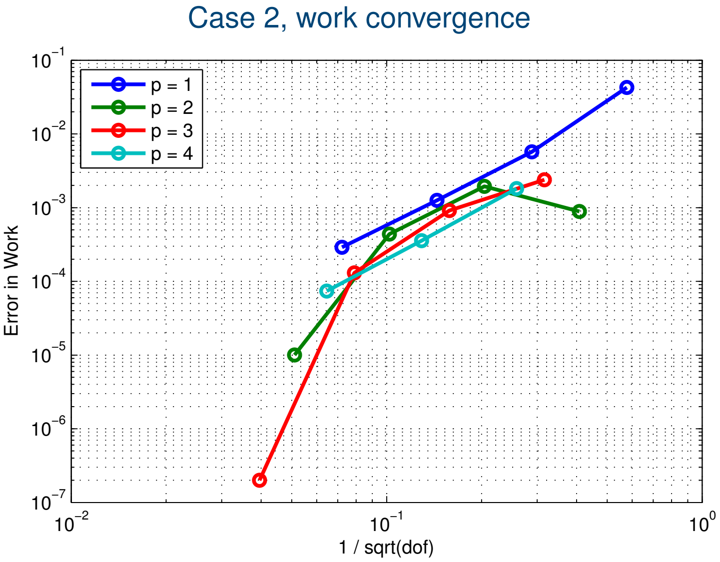

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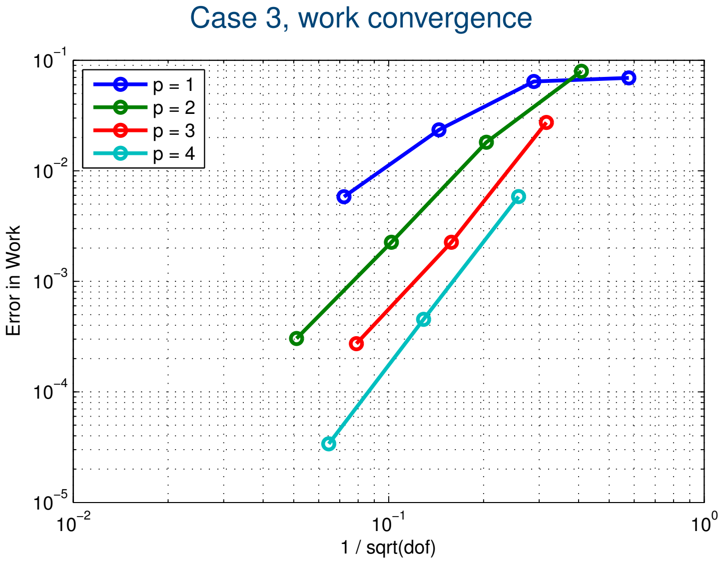

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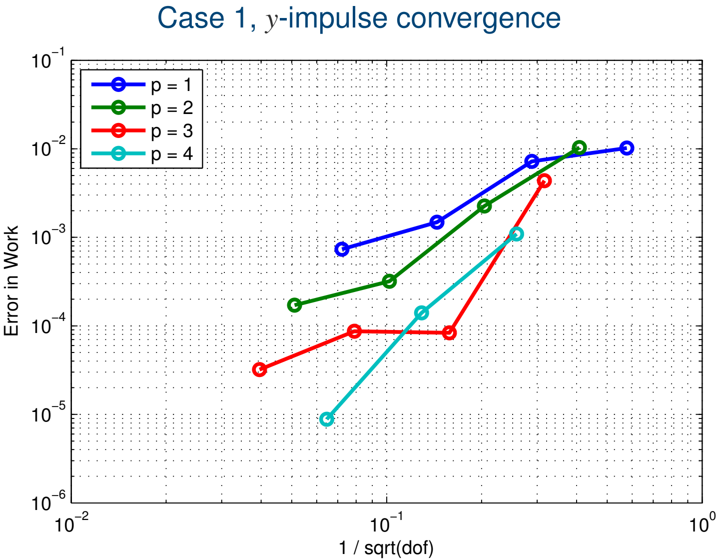

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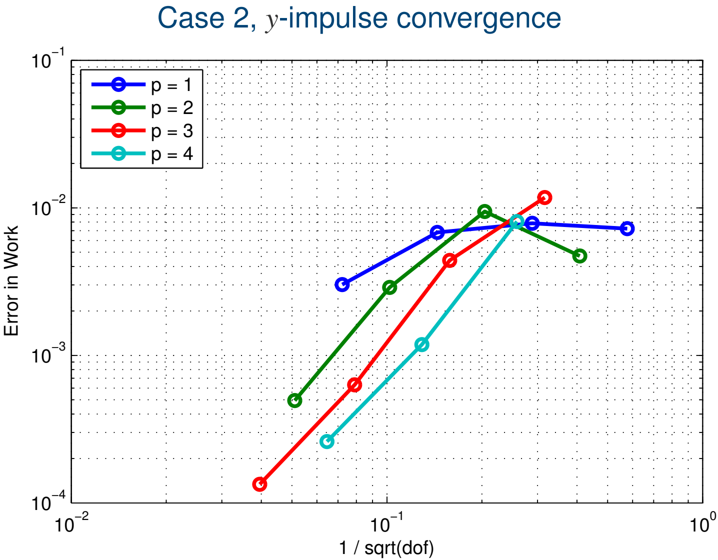

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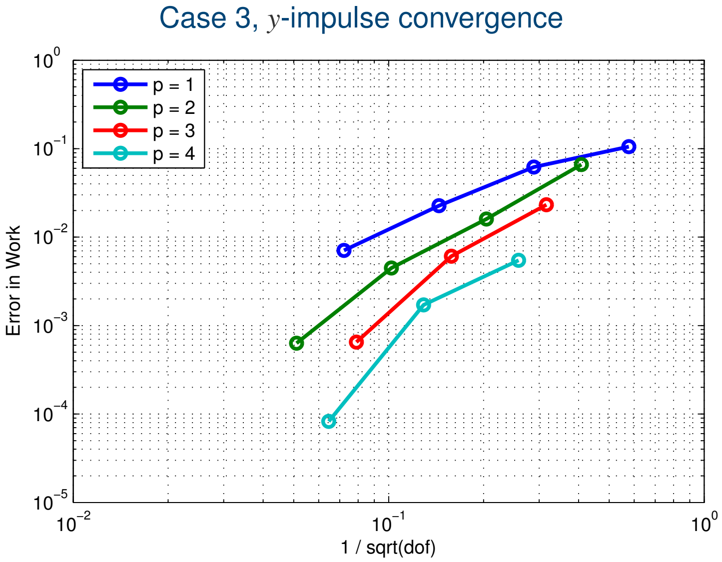
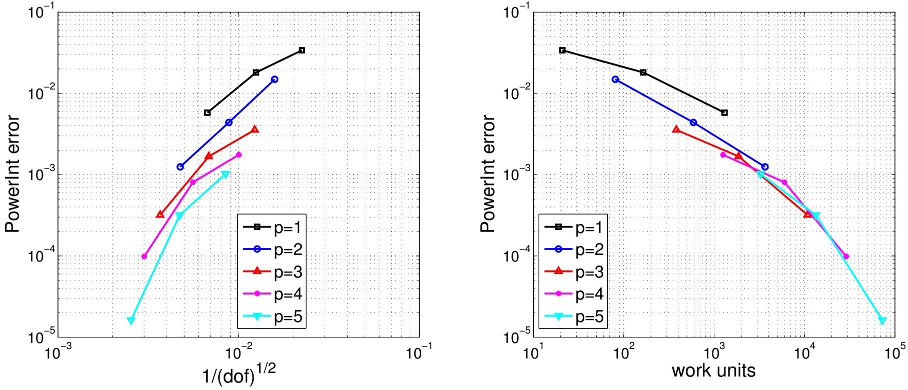


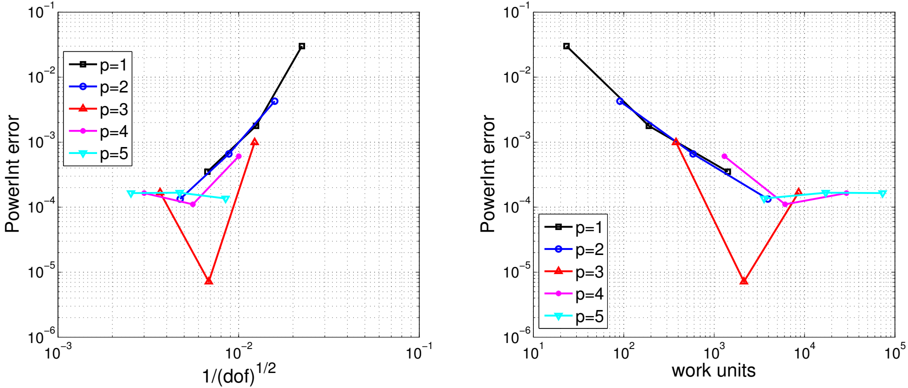
U. Michigan Results – Convergence

Case 1, work convergence

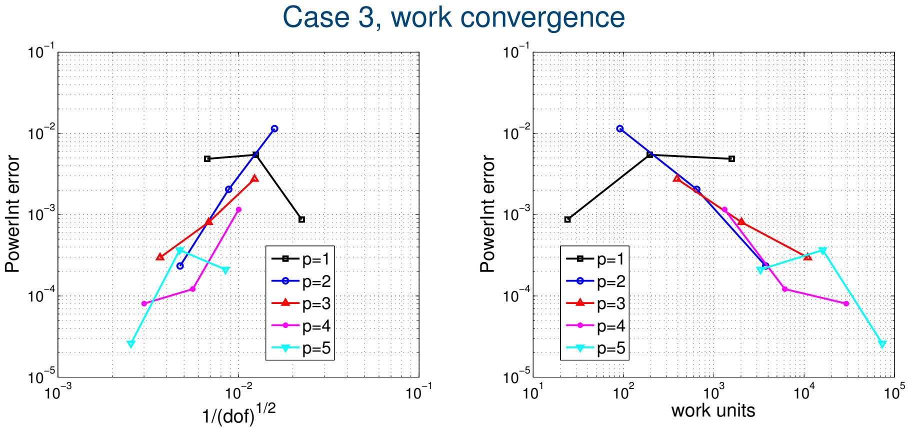


U. Michigan Results – Convergence

Case 2, work convergence

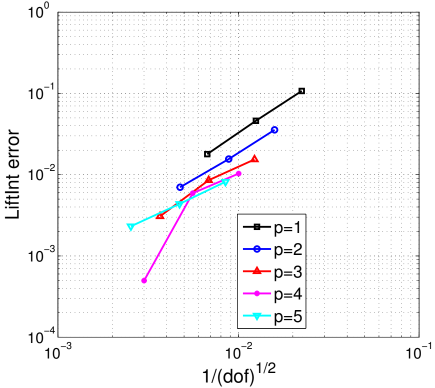


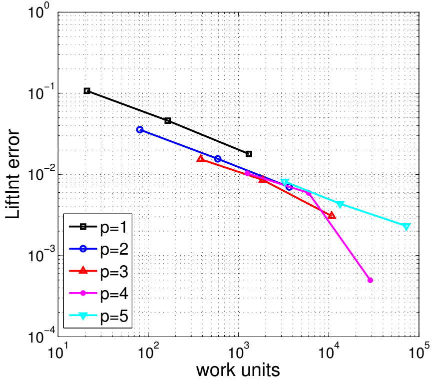
U. Michigan Results – Convergence



U. Michigan Results – Convergence

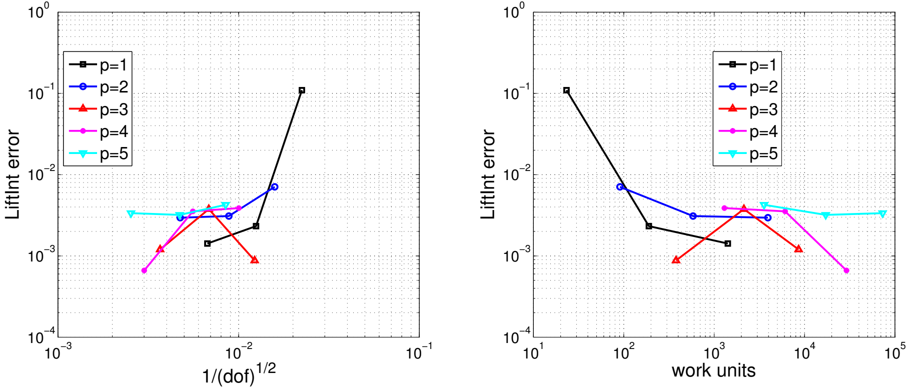
Case 1, y-impulse convergence





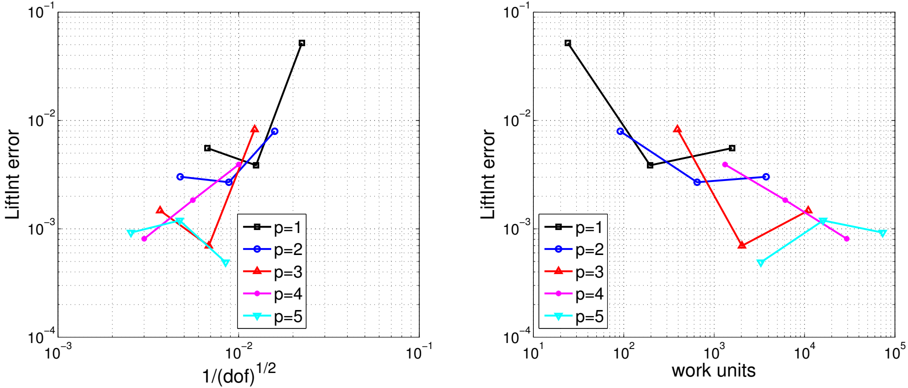
U. Michigan Results – Convergence

Case 2, y-impulse convergence



U. Michigan Results – Convergence

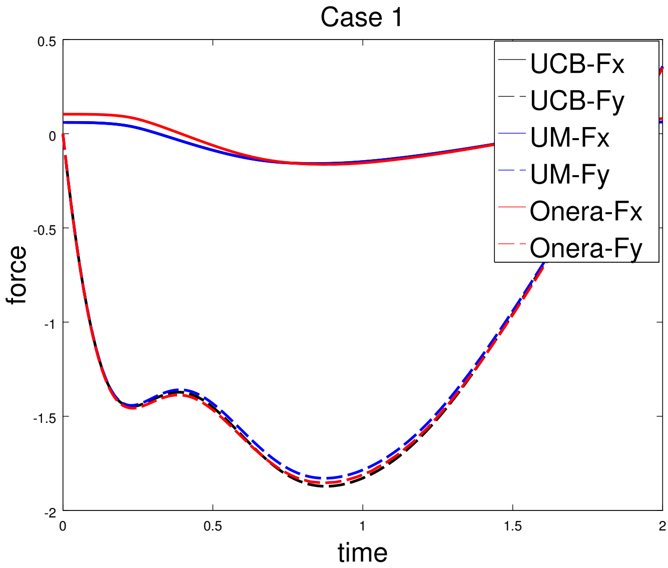
Case 3, y-impulse convergence



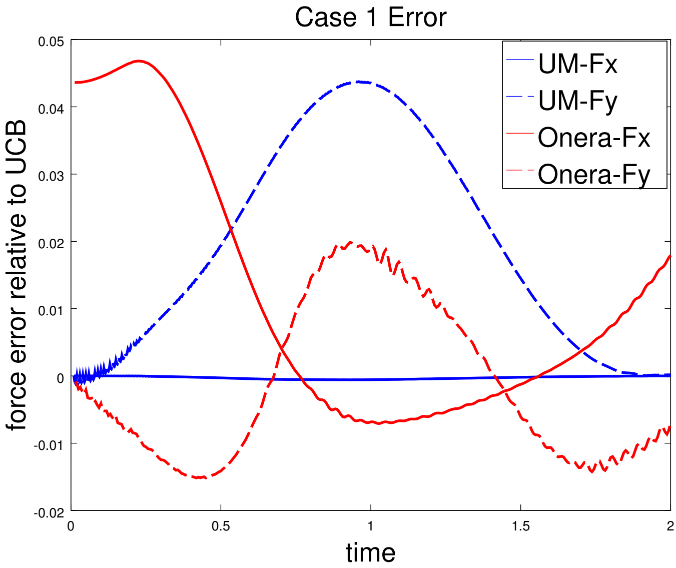
Participating Groups – Results

	UC Berkeley	U. Michigan	Onera
Scheme	DG	DG	FV
Degree p	1,2,3,4	1,2,3,4,5	2,4
Time-stepping	DIRK3 impl.	ESDIRK5 impl.	BDF3 impl. dual ts
ALE motion	rigid	blended	blended
Steady x-force	0.06000705055	0.06000682777	0.1028
Case 1, work	-1.40945	-1.38350	-1.40493
Case 1, impulse	-2.37606	-2.33125	-2.37286
Case 2, work	-0.22029	-0.20490	-0.20788
Case 2, impulse	0.58978	0.61004	0.59123
Case 3, work	0.40089	0.36377	0.33388
Case 3, impulse	1.69529	1.67047	1.70758

Group comparison – Forces



Forces, differences with UC Berkeley Results



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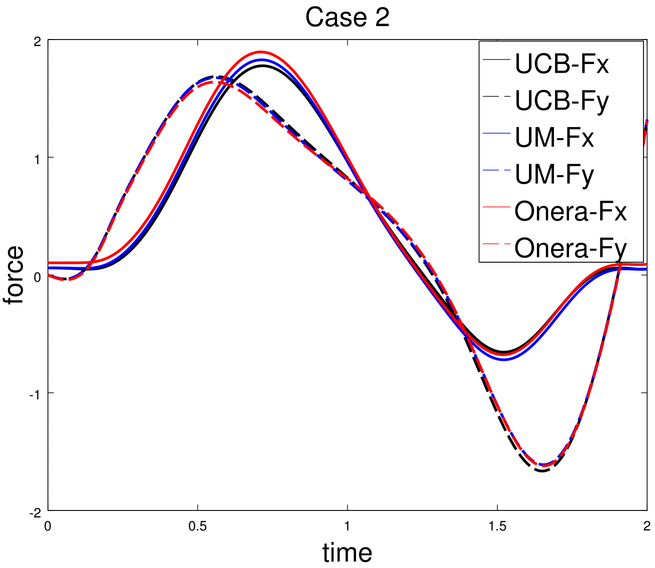

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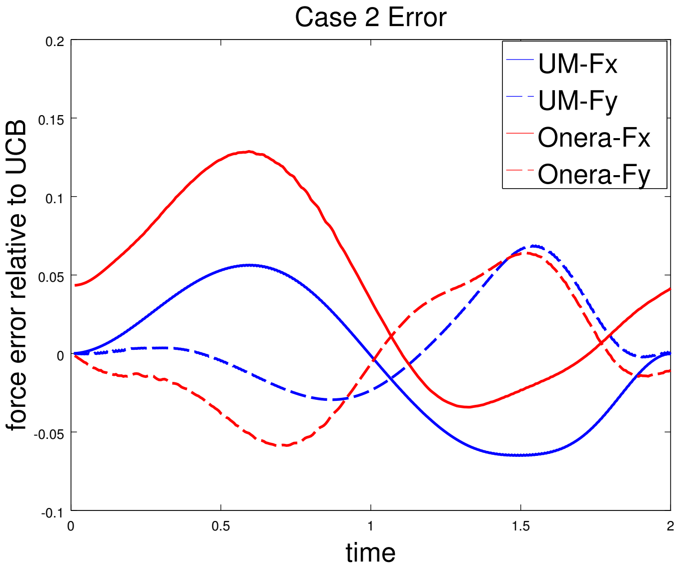

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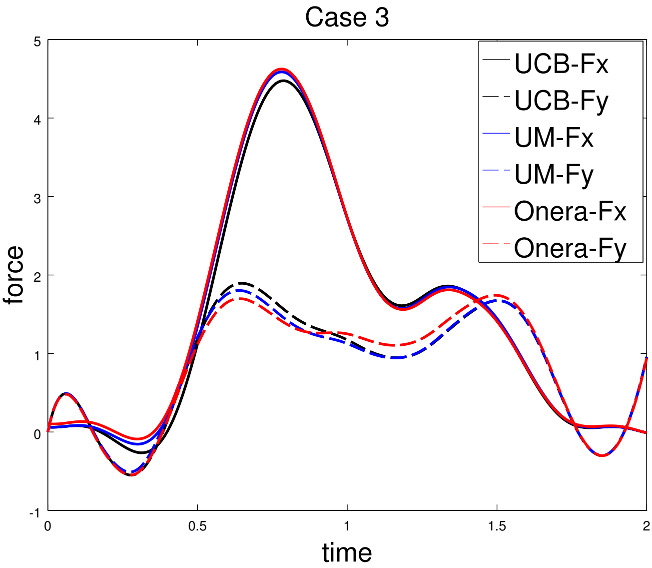

Forces, differences with UC Berkeley Results