# *On the implementation of X-LES in a high-order implicit DG solver*

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**Towards Industrial LES/DNS in Aeronautics**
**Paving the Way for Future Accurate CFD**
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# *Brief code summary*

- Discontinuous Galerkin (DG) method on hybrid grids
- Physical frame orthonormal basis functions
- 2D/3D steady and unsteady compressible and incompressible flows
- Explicit and implicit time accurate integration
- Fixed or rotating frame of reference

- Euler
- Navier–Stokes
- RANS coupled with the k-ω (EARSM)
- Hybrid RANS/LES (X-LES)

- MPI parallelism
- Fortran language

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- Physical frame orthonormal basis functions
- 2D/3D steady and unsteady compressible and incompressible flows
- Explicit and implicit time accurate integration - *variables*
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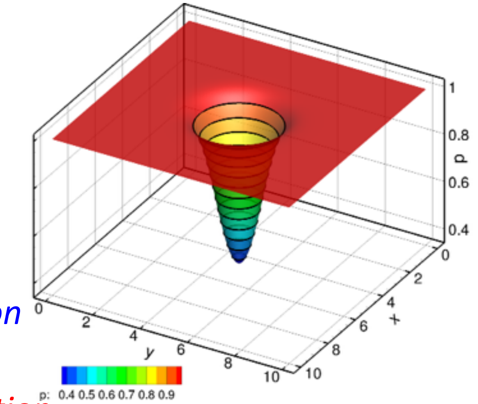
# *Implicit accurate time integration*

Several high-order temporal schemes are implemented

- Modified Extended BDF
- Two Implicit Advanced Step-point (TIAS)
- Explicit Singly Diagonally Implicit R-K (ESDIRK)
- linearly implicit Rosenbrock method

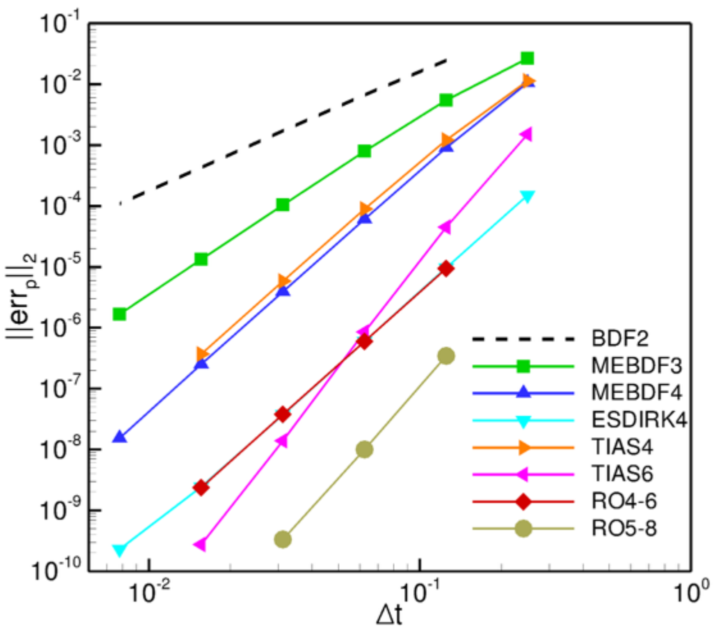
} *non-linear systems solution*

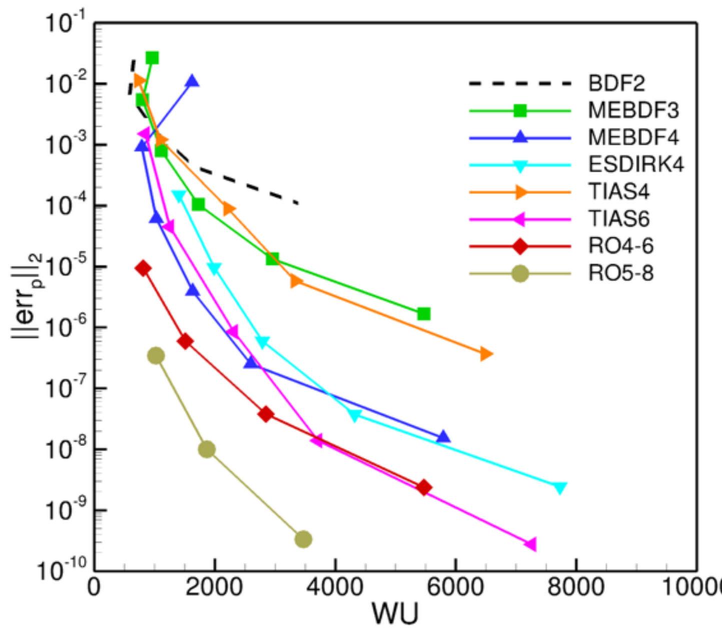
*linear systems solution (here via GMRES)*



*i)* Hi-O schemes are more efficient than Lo-O ones for high required accuracy

*ii)* Rosenbrock-type schemes are appealing both for accuracy and efficiency




Convection of an isentropic vortex $P^6$ solution on 50X50 el.

**Bassi, et al., "Investigation of high-order temporal schemes for the discontinuous Galerkin solution of the Navier- Stokes equations", proceedings of 11th WCCM 2014, 5th ECCM 2014, 6th ECFD 2014 (2014) pp. 5651–5662**

# *Rosenbrock schemes in a nutshell (I/II)*

From the DG spatial discretization we obtain a system of non-linear ODEs or DAEs

$$\mathbf{M_P}\left(\mathbf{W}\right)\frac{d\mathbf{W}}{dt}+\mathbf{R}\left(\mathbf{W}\right)=\mathbf{0} \qquad \widetilde{\mathbf{R}}=\mathbf{M_P}^{-1}\mathbf{R}$$

$$\mathbf{W}^{n+1}=\mathbf{W}^{n}+\sum_{j=1}^{s}m_j\mathbf{Y}_j$$

$$\left(\frac{\mathbf{M_P}}{\gamma\Delta t}+\mathbf{J}-\frac{\partial\mathbf{M_P}}{\partial\mathbf{W}}\widetilde{\mathbf{R}}\right)^{n}\mathbf{Y}_i=-\mathbf{M}_\mathbf{P}^{n}\left[\widetilde{\mathbf{R}}\left(\mathbf{W}^{n}+\sum_{j=1}^{i-1}a_{ij}\mathbf{Y}_j\right)-\sum_{j=1}^{i-1}\frac{c_{ij}}{\Delta t}\mathbf{Y}_j\right]$$

$$i=1,\ldots,s$$

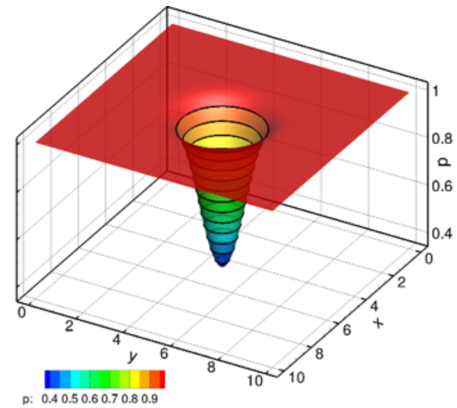
only a linear system need to be solved for each stage

*i.e.* the Jacobian $\mathbf{J}=\partial\mathbf{R}/\partial\mathbf{W}$ is assembled and factored only once per time step

With orthonormal basis functions (*physical space*) $\mathbf{M_P}$ reduces to the identity for compressible flows with conservative variables

For other sets of variables their DOFs can be coupled within $\mathbf{M_P}$ thus resulting in a matrix which can not be diagonal

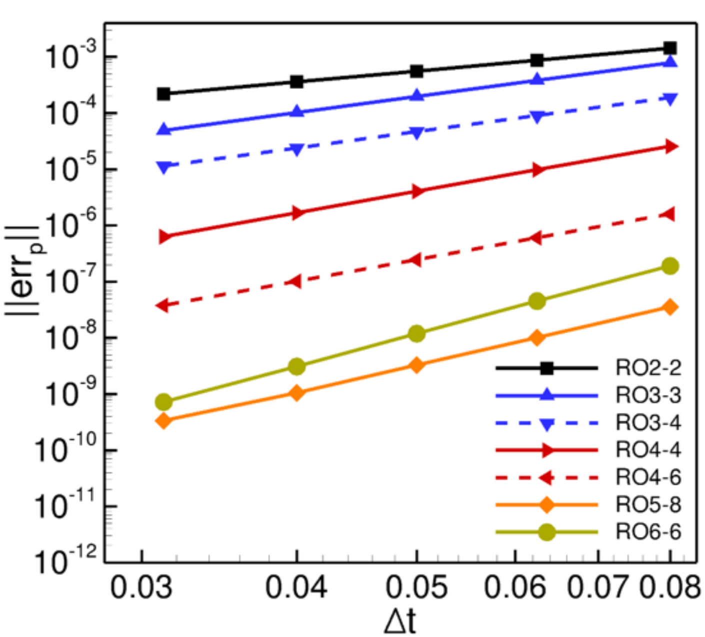
# *Rosenbrock schemes in a nutshell (II/II)*

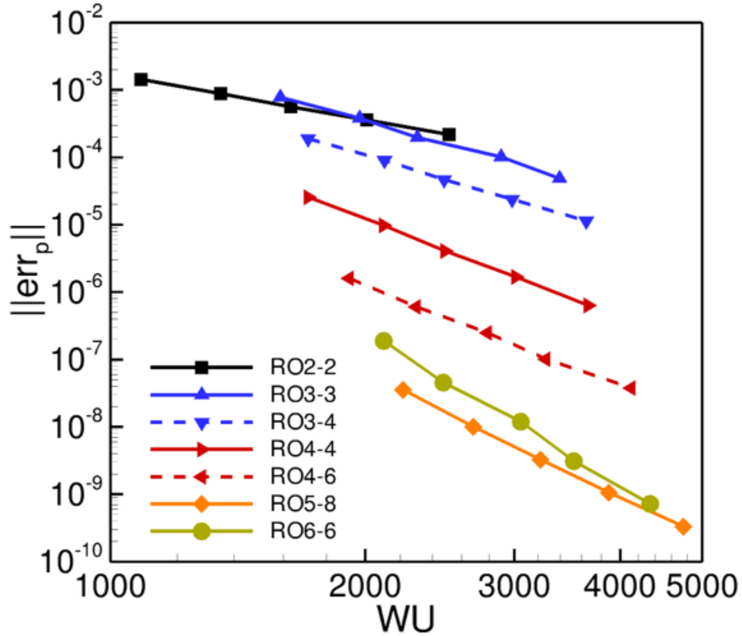



Several Rosenbrock schemes, from order two to order six, have been compared

No need to "exactly" solve systems: GMRES tolerance can be increased with confidence with a significant reduction of WU

For a given order of accuracy, among the schemes considered, those with more stages are more accurate and efficient, *e.g.* RO5-8 vs. RO6-6




Convection of an isentropic vortex $P^6$ solution on 50X50 el.

**Bassi et al. "Linearly implicit Rosenbrock-type Runge-Kutta schemes for the Discontinuous Galerkin solution of compressible and incompressible unsteady flows" Computers & Fluids, 118 (2015) pp. 305 – 320**

# *Working variables*

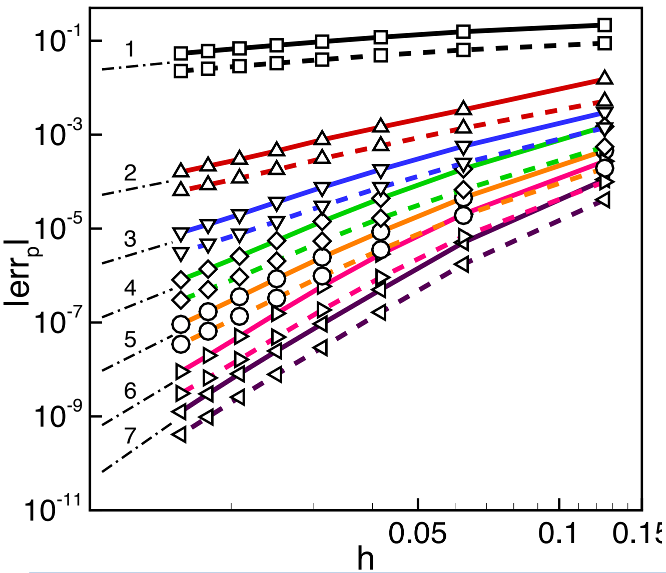
Primitive variables to ensure the positivity of all thermodynamic variables…

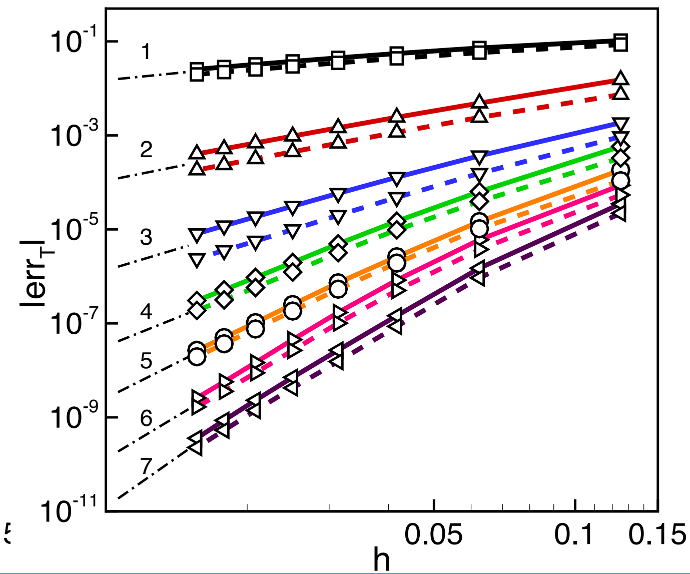
We work with polynomial approximations not directly for $p$ and $T$ but for their logarithms $\widetilde{p} = log(p)$ and $\widetilde{T} = log(T)$

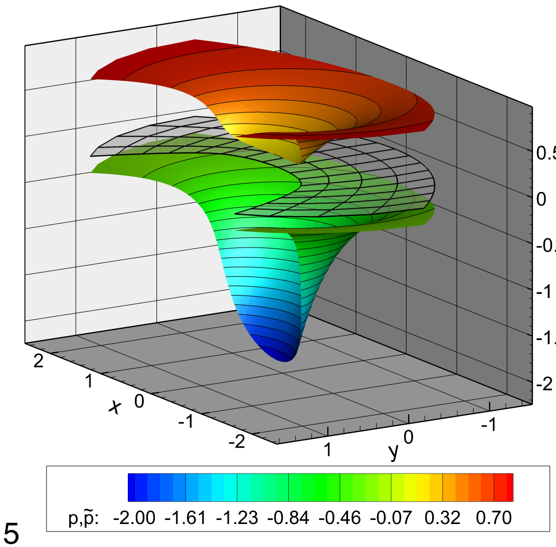
$$\widetilde{p}(\mathbf{x}) = \phi_j(\mathbf{x}) W_j^{\widetilde{p}}, \quad \widetilde{T}(\mathbf{x}) = \phi_j(\mathbf{x}) W_j^{\widetilde{T}}, \quad j = 1, \cdots, N_{dof}^k$$

In this way the computed values $p = e^{\widetilde{p}}$ and $T = e^{\widetilde{T}}$ are always positive

Improved code robustness with a low implementation effort $(\mathbf{M_P})$




Bassi et al. "Linearly implicit Rosenbrock-type Runge-Kutta schemes for the Discontinuous Galerkin solution of compressible and incompressible unsteady flows" Computers & Fluids, 118 (2015) pp. 305 – 320

# *eXtra-Large Eddy Simulation (X-LES) in a nutshell (I/II)*

## Pros

- hybrid RANS\LES formulation independent from the wall distance
- use in LES mode of a clearly defined SGS based on the k-equation
- use of a k-ω turbulence model integrated to the wall

## Cons

the *filter width* parameter is often related to the local element size

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho u_j k) = \frac{\partial}{\partial x_j}\left[(\mu + \sigma^* \overline{\mu}_t)\frac{\partial k}{\partial x_j}\right] + P_k - D_k$$

$$\frac{\partial}{\partial t}(\rho\widetilde{\omega}) + \frac{\partial}{\partial x_j}(\rho u_j \widetilde{\omega}) = \frac{\partial}{\partial x_j}\left[(\mu + \sigma\overline{\mu}_t)\frac{\partial \widetilde{\omega}}{\partial x_j}\right] + (\mu + \sigma\overline{\mu}_t)\frac{\partial \widetilde{\omega}}{\partial x_k}\frac{\partial \widetilde{\omega}}{\partial x_k} + P_\omega - D_\omega + C_D$$

*…an "original" interpretation for the X-LES implementation…*

**Bassi et al. "Time Integration in the Discontinuous Galerkin Code MIGALE - Unsteady Problems"**
**In IDIHOM: Industrialzation of High-Order Methods - A Top-Down Approach, Vol. 128 of Notes on Numerical Fluid Me- chanics and Multidisciplinary Design Springer International Publishing**

## *eXtra-Large Eddy Simulation (X-LES) in a nutshell (II/II)*

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho u_j k) = \frac{\partial}{\partial x_j}\left[(\mu + \sigma^* \overline{\mu}_t)\frac{\partial k}{\partial x_j}\right] + P_k - D_k$$

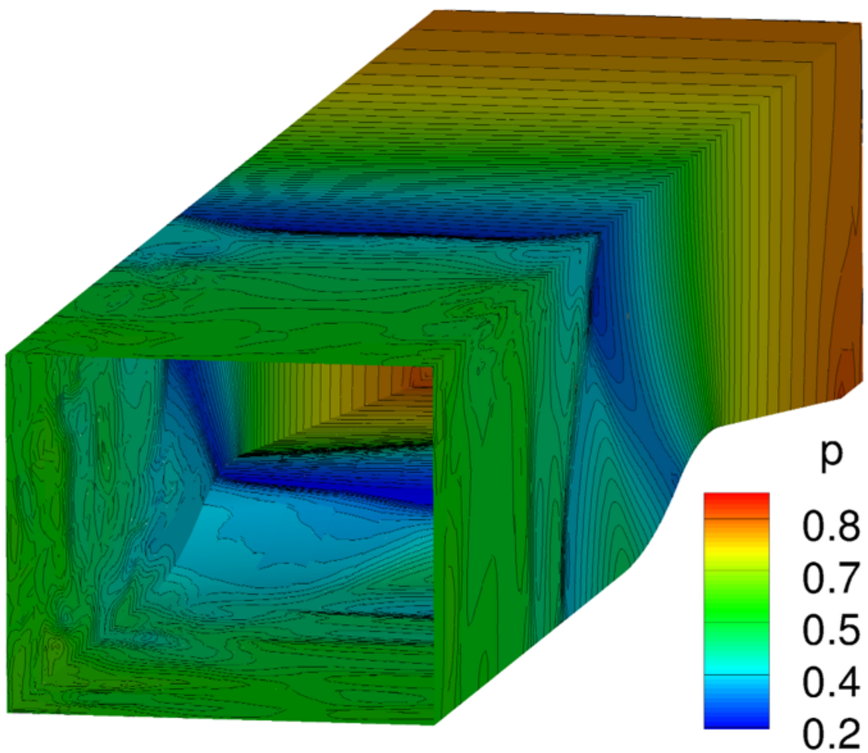
$$\overline{\mu}_t = \alpha^* \frac{\rho \overline{k}}{\hat{\omega}} \qquad D_k = \beta^* \rho \overline{k} \hat{\omega} \qquad \overline{k} = \max(0, k)$$

$$\hat{\omega} = \max\left(e^{\widetilde{\omega}_r}, \frac{\sqrt{\overline{k}}}{C_1 \Delta}\right)$$

| | RANS | LES | ILES |
|---|---|---|---|
| $\overline{\mu}_t$ | $\alpha^* \frac{\rho \overline{k}}{e^{\widetilde{\omega}_r}}$ | $\alpha^* \rho \sqrt{\overline{k}} C_1 \Delta$ | 0 |
| $D_k$ | $\beta^* \rho \overline{k} e^{\widetilde{\omega}_r}$ | $\beta^* \rho \frac{\overline{k}^{\frac{3}{2}}}{C_1 \Delta}$ | 0 |

# *X-LES of a shock BL interaction on a swept bump (AR2 of HiOCFD, TC-F3 of TILDA)*

$P^2$ converged computations with RANS+k-ω (also in its low-Re version) and EARSM1 have been performed and used as initialization for X-LES




72960 hexahedral elements

- Inlet boundary conditions
  - $p_{0i}$ = 92000Pa
  - $T_{0i}$ = 300K
  - $Re_H = 1.69 \times 10^6$
- Outlet static pressure used to impose the shock position *(model dependent!)*
- LBE(RO1-1) to quickly find the “right” pressure ratio
- RO3-3 for the time-accurate solution
- Filter width Δ = 5e-2 *(strong influence!)*
- parallel computation on 96 cores

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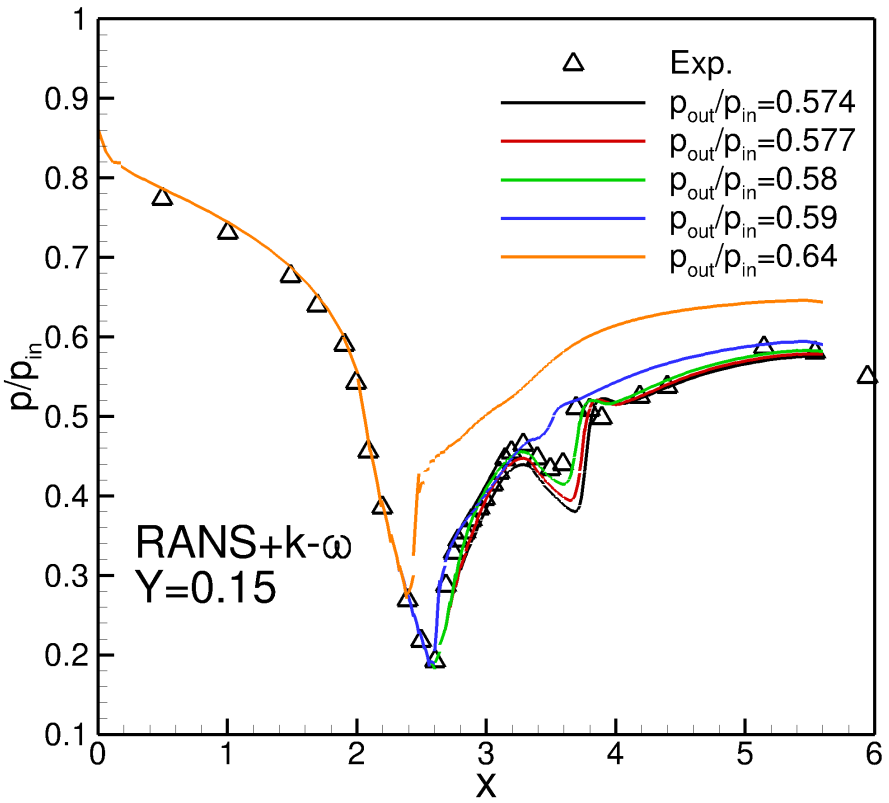


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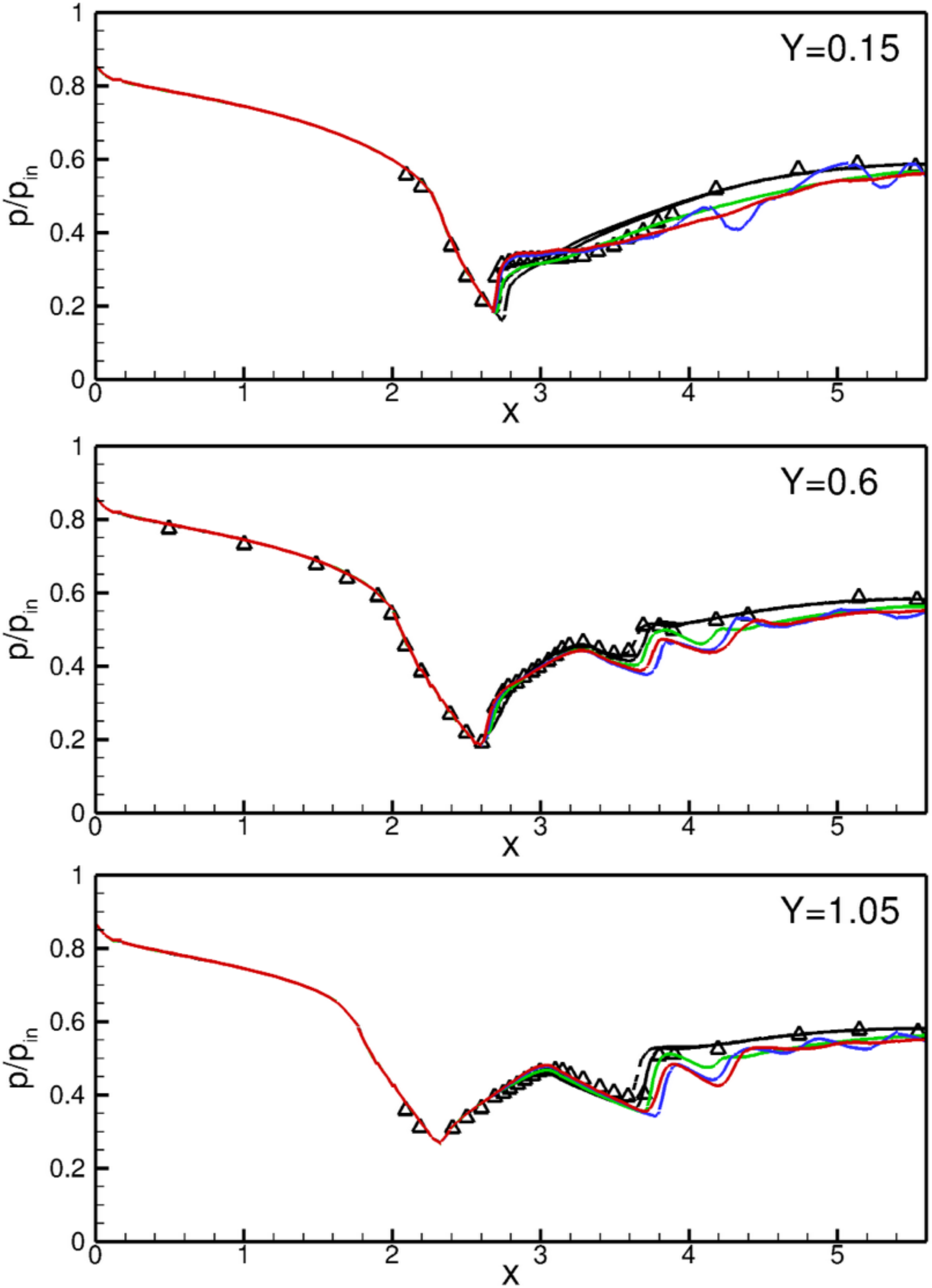
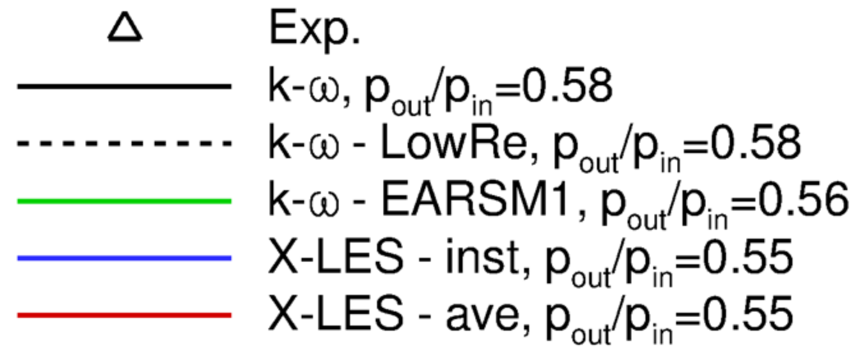
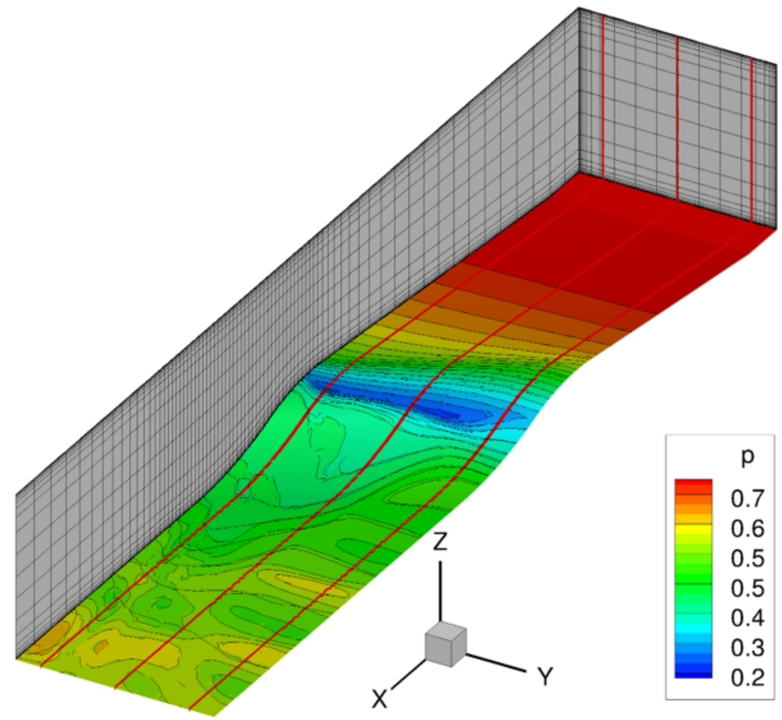
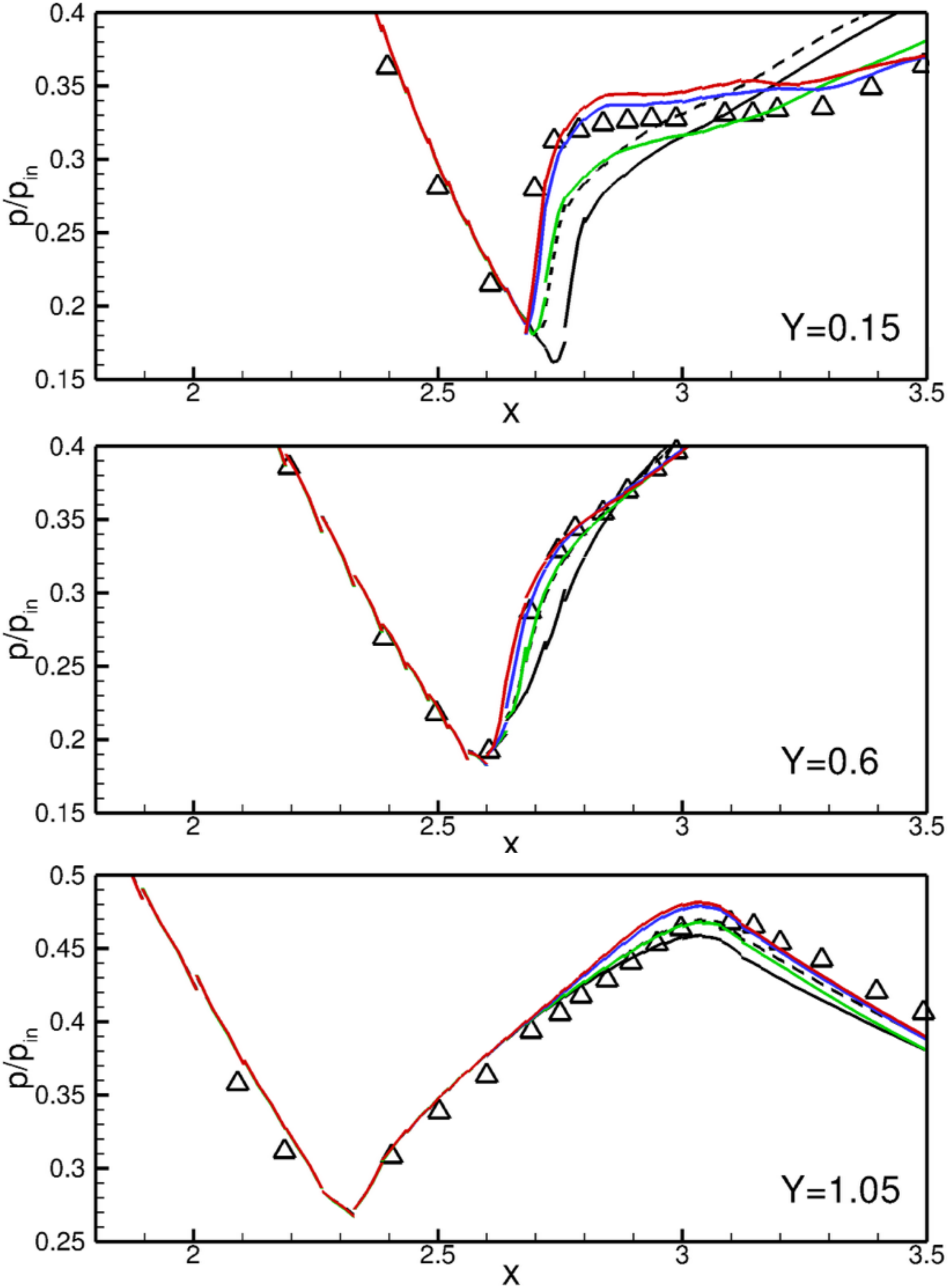
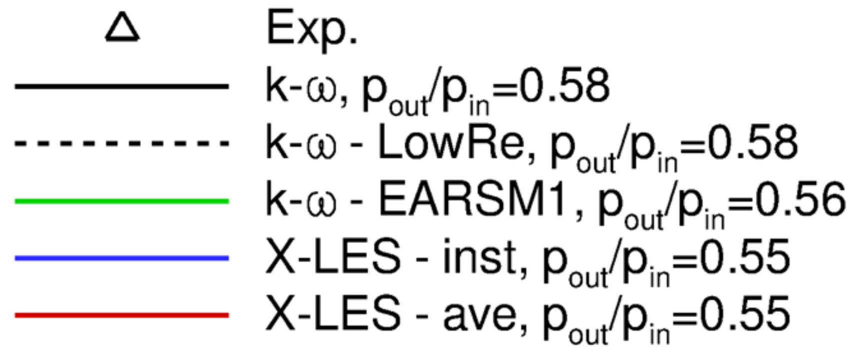


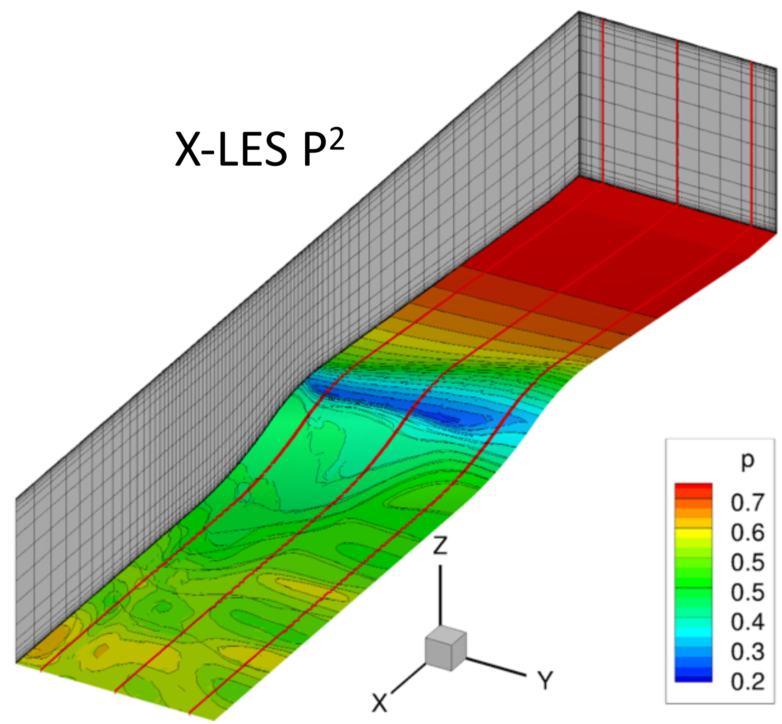




The profiles are in reasonable agreement with the experiments and the numerical results of Cahen et al.

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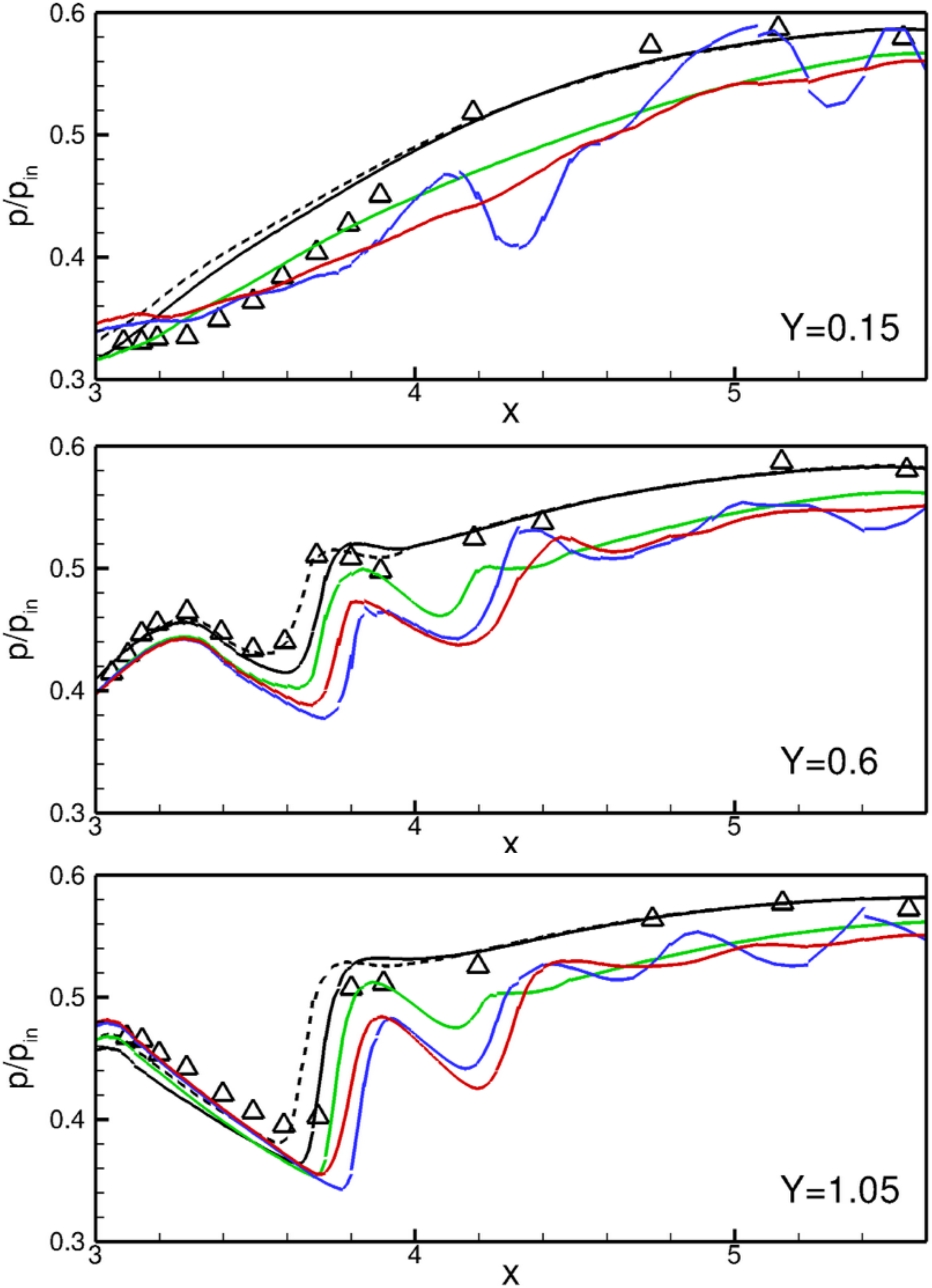


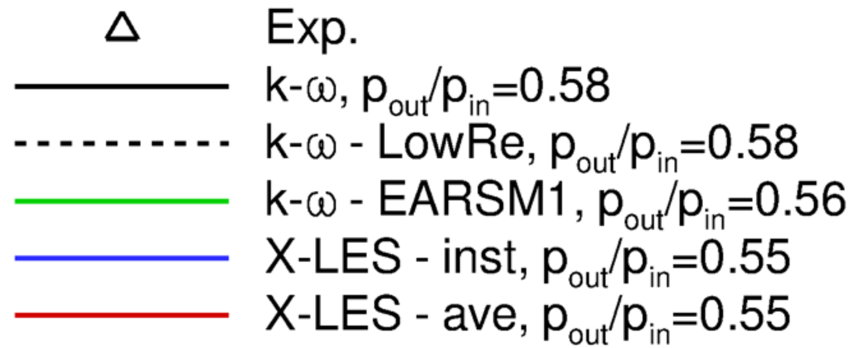


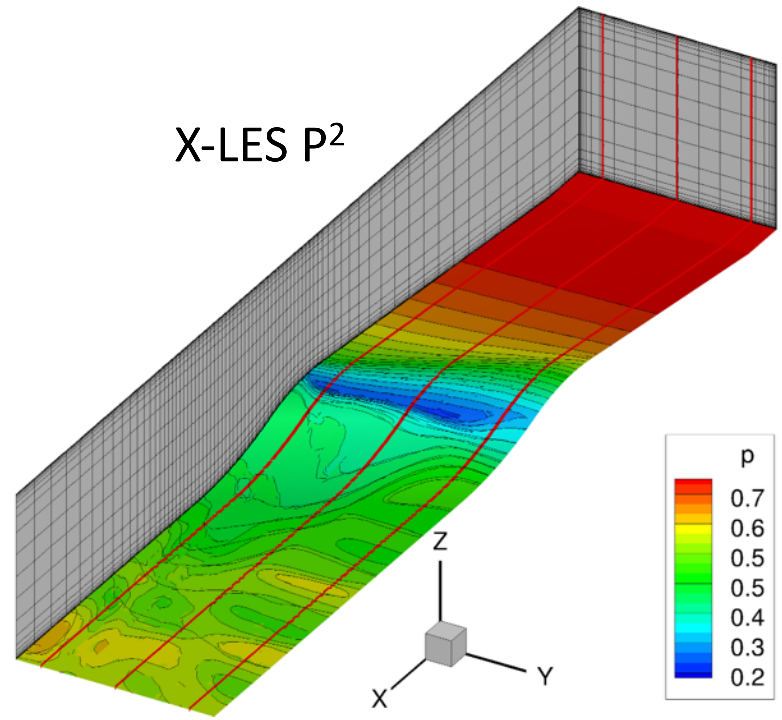


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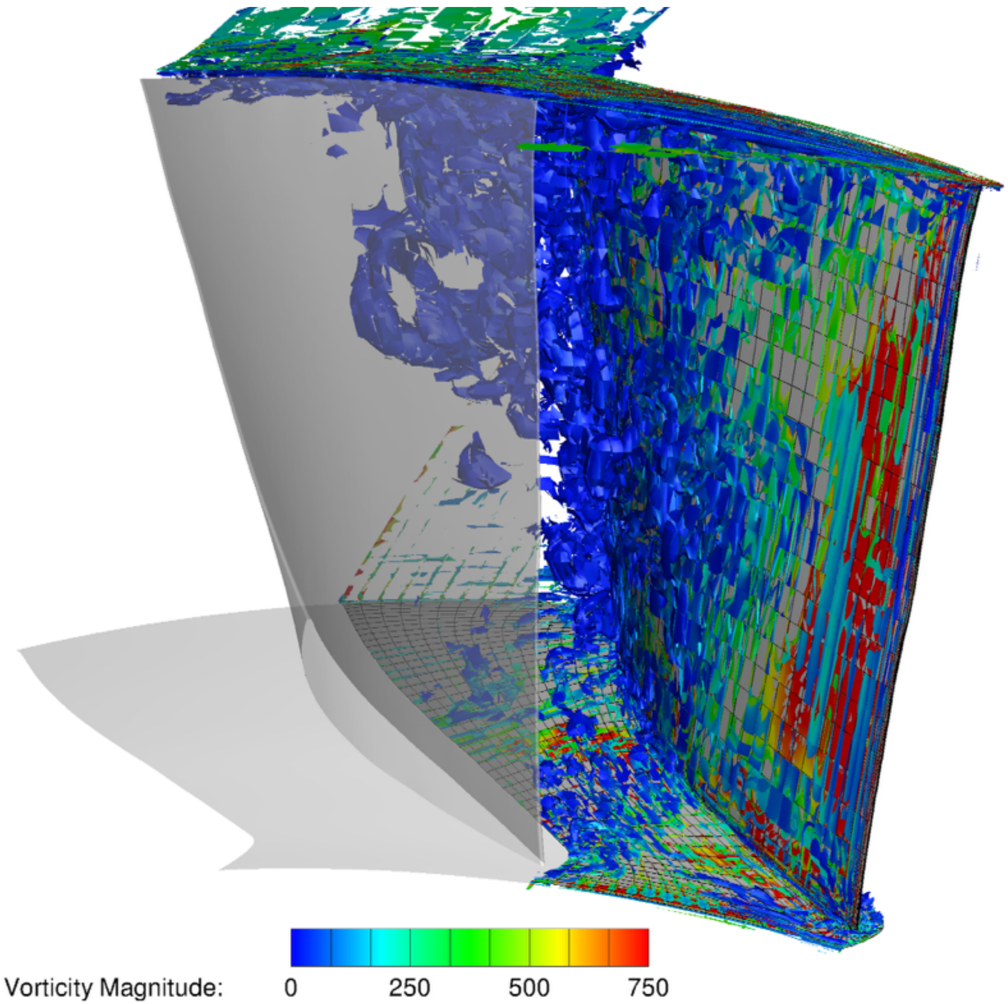
# *X-LES of the transonic flow field in the NASA Rotor 37 (TC-P6 of TILDA)*




iso-surface of Q-criterion
160512 hexahedral elements

- $P^2$ computation using RO3-3
- Filter width Δ=5e-5
- Boundary conditions
  - $p_{01}$ = 101325Pa
  - $T_{01}$ = 288K
  - ω= 1800rad/s
  - $Tu_1$ = 3%
  - $\alpha_1$ = 0°
- parallel computation on 96 cores

According to our first experiences, for a practical usage of X-LES, initializing with RANS seems mandatory

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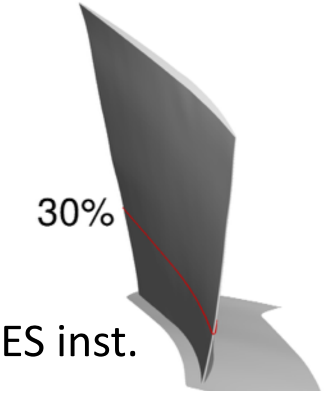
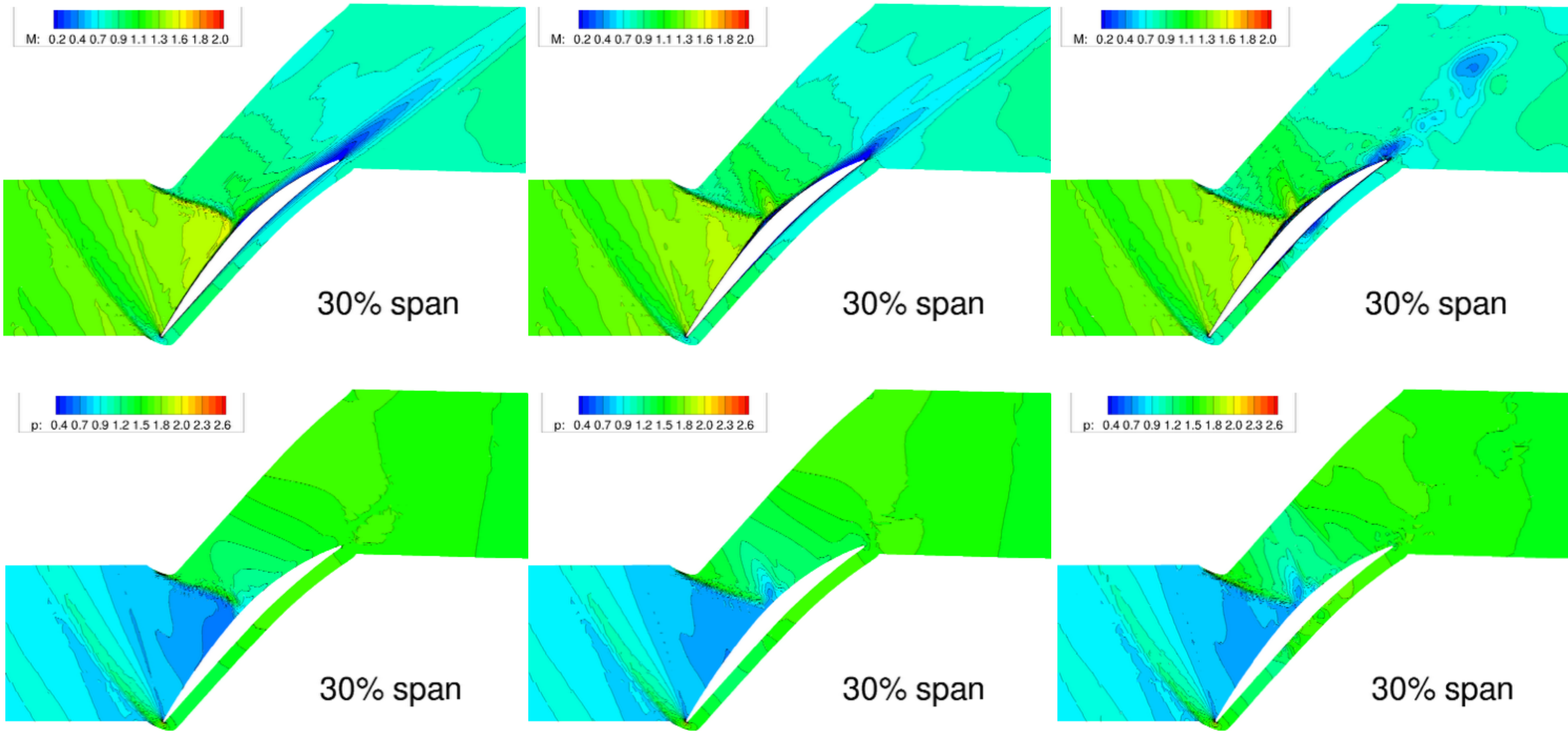



RANS X-LES ave. X-LES inst.




**Bassi et al. "Assessment of a high-order accurate Discontinuous Galerkin method for turbomachinery flows"** ***Accepted for publication on*** **International Journal of CFD (2016)**

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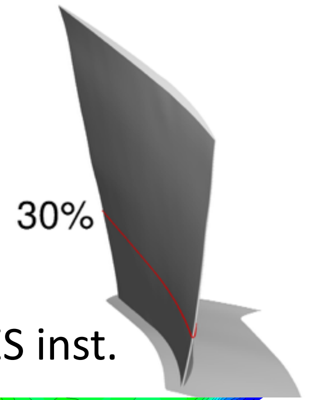
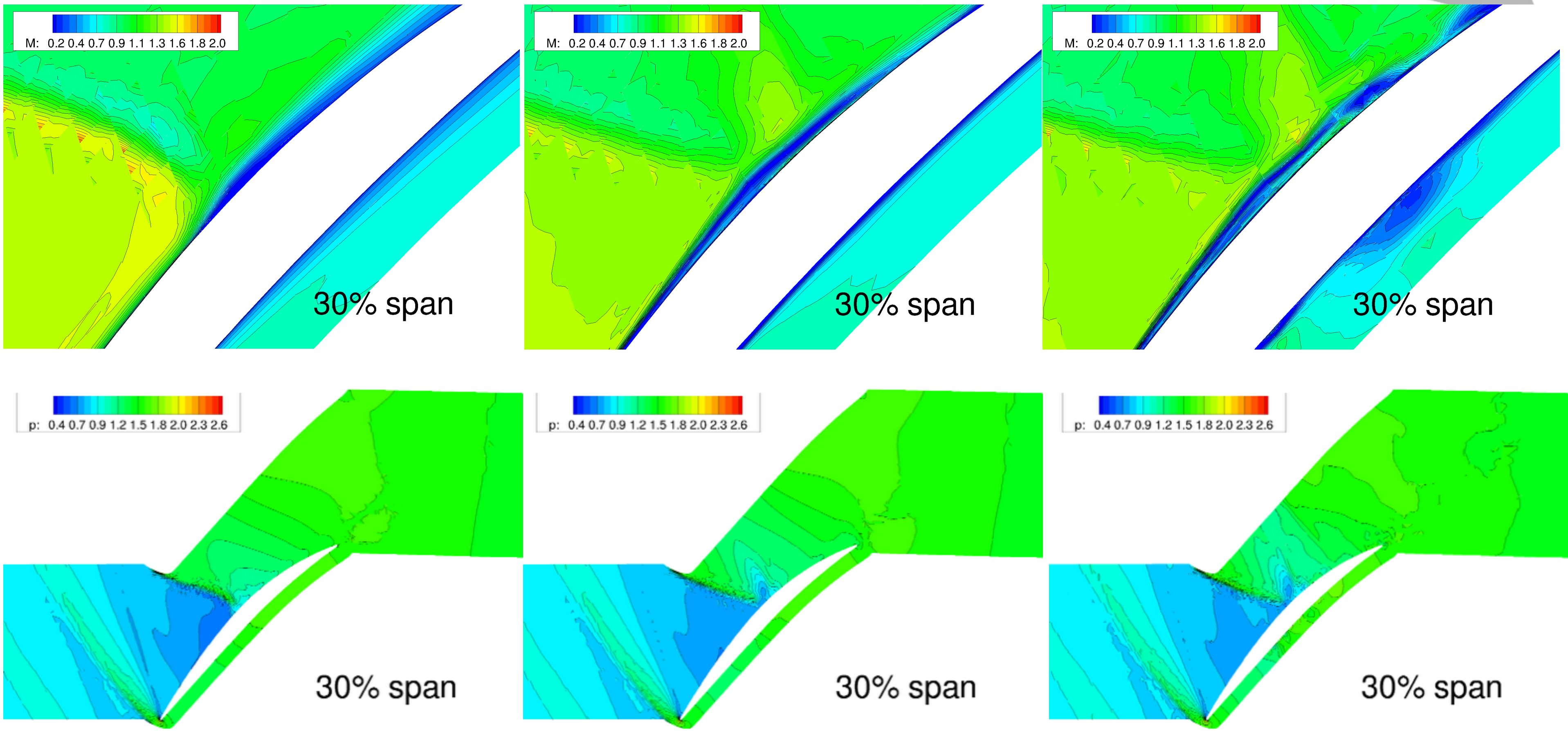



RANS | X-LES ave. | X-LES inst.

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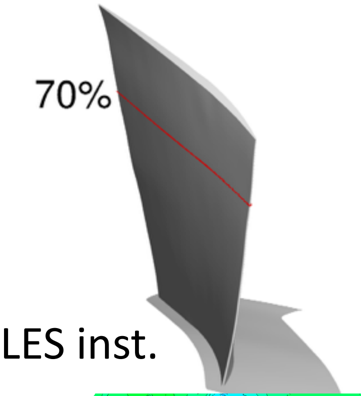


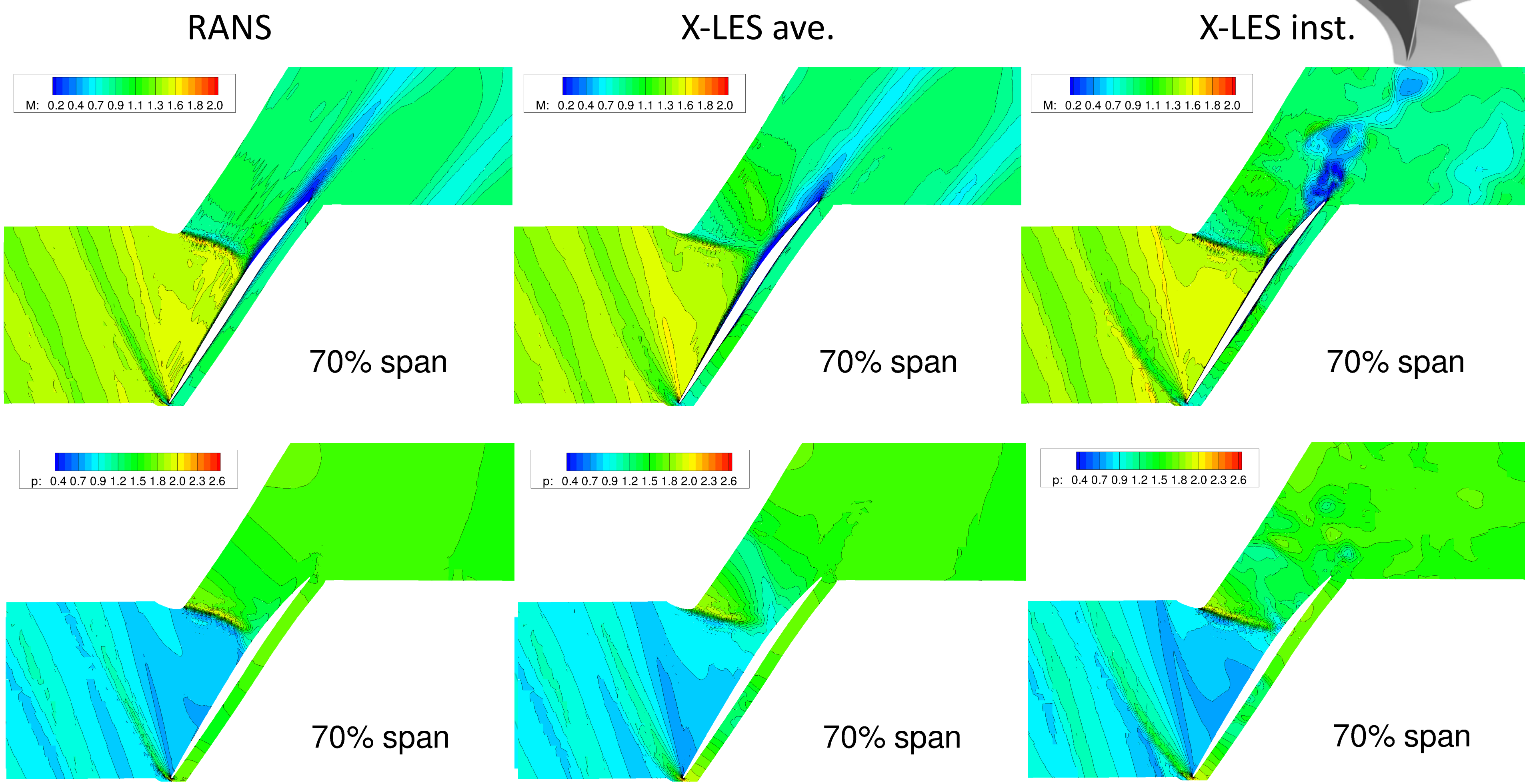


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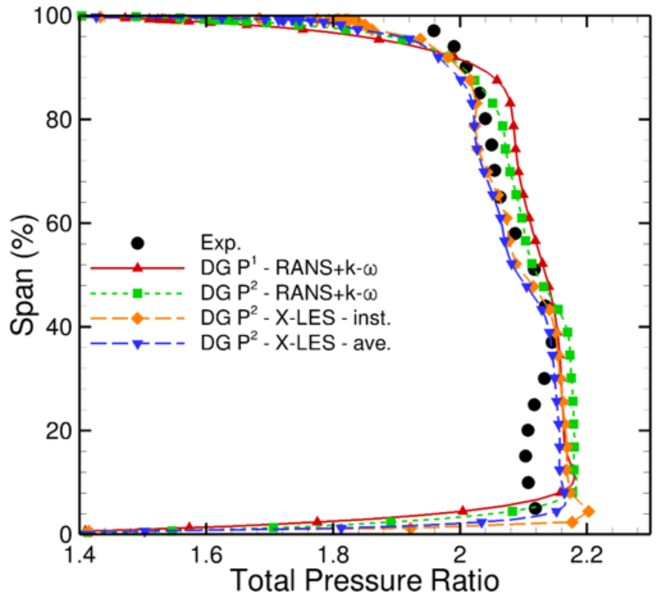
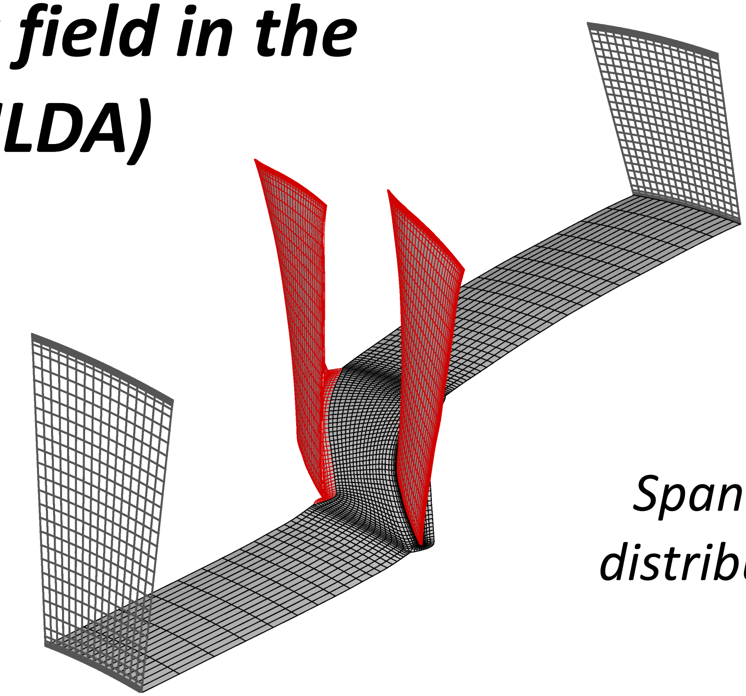
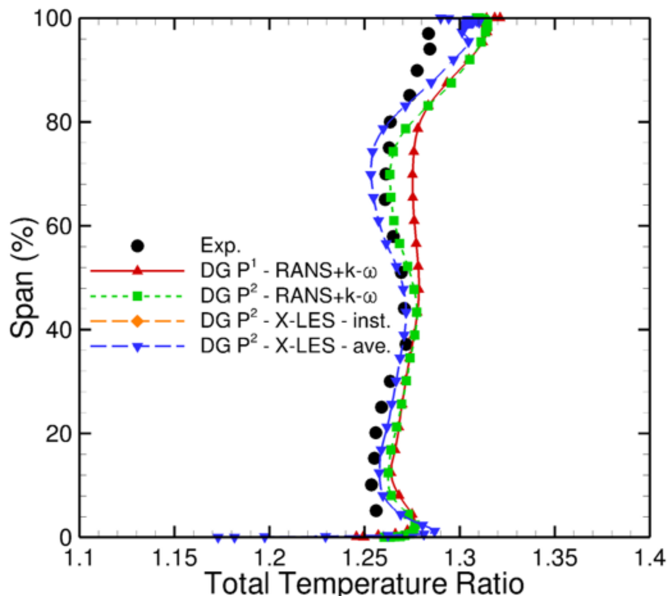
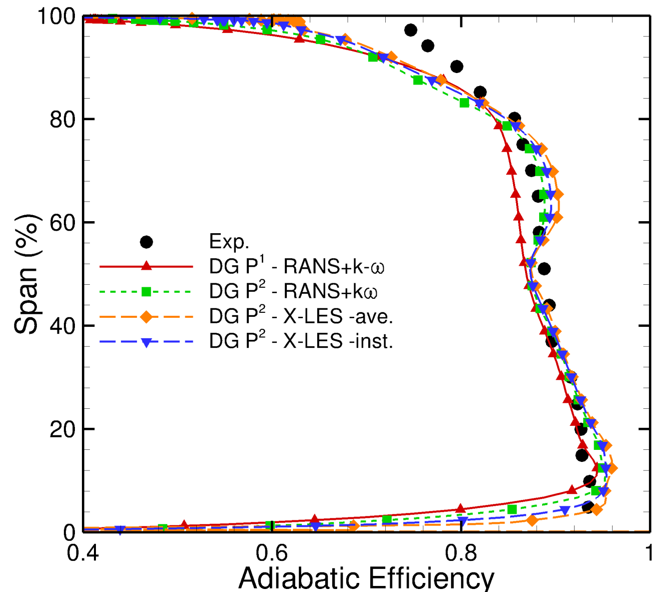


*Spanwise distributions*



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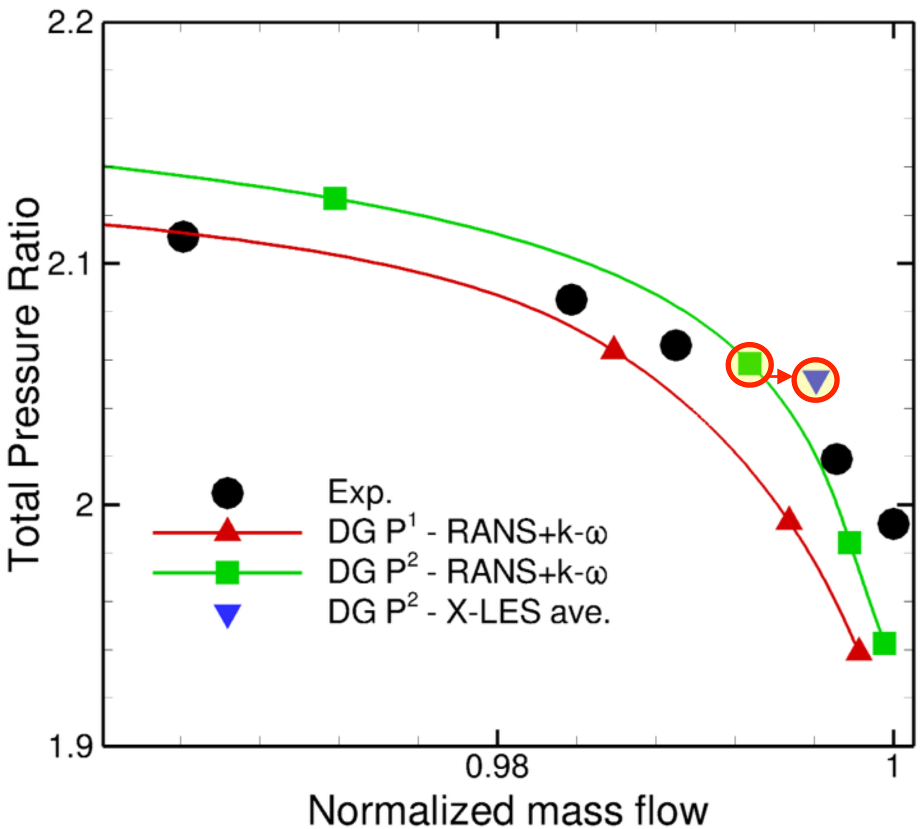


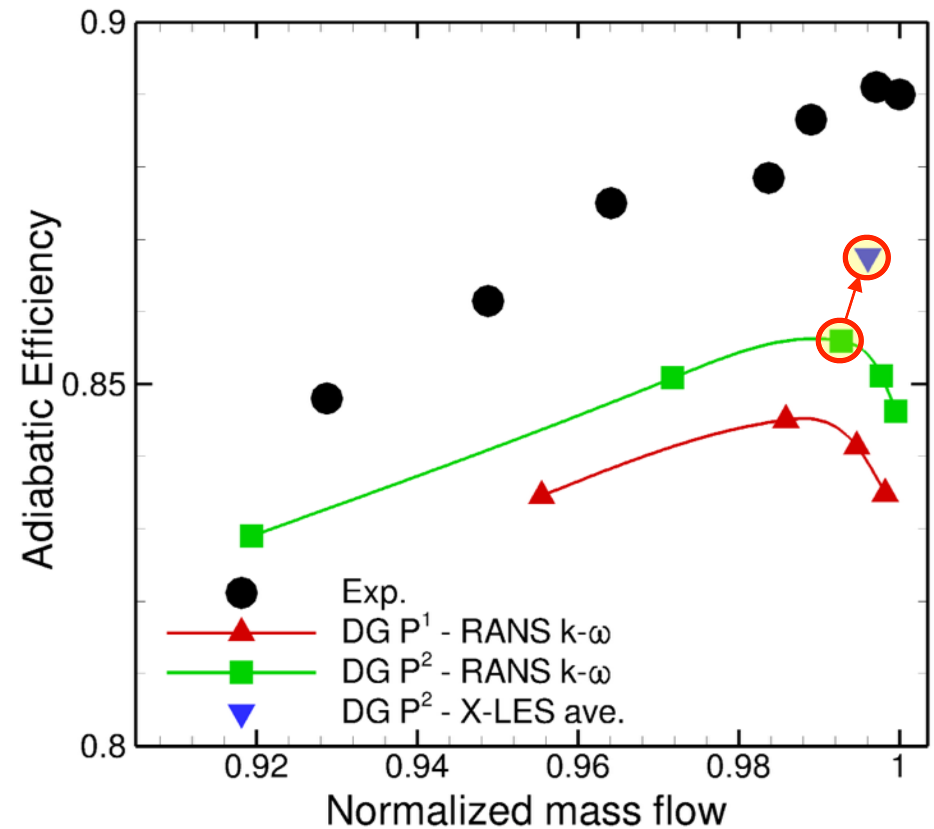


Initializing the solution from a flow field corresponding
to a normalized mass flow, resulting from a RANS computation, of ≈0.99
X-LES moved towards ≈0.996

## *Conclusions…*

- First results of an implicit DG solution of X-LES have been presented
- Implicit time accurate integration coupled with X-LES has shown to be robust

## *…and future work*

- Compute the solution on a finer grid and\or a finer polynomial degree
- The influence on the solution of filter width parameter (Δ) need further investigation
- Evaluation of the numerical dissipation of the solver by computing the Decay of Isotropic Turbulence (DIT) test case