NXO
a Spatially High Order Finite Volume Numerical Method for Compressible Flows

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ONERA
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return on innovation

Basics of the NXO scheme

- Euler or Navier-Stokes for perfect gas law of state
- 1 dof per cell and per equation (Volume average), wide stencils,
- Coarse partitioning of the grid : one partition per node, shared memory hybrid programming (MPI / OpenMP or Cuda)

- Polynomial Reconstruction algorithm for the conservative variables ***or flux density fields***

➔ *Preprocessor phase : Weighted Least-Square polynomial degree adapts to the "quality" of the stencils*

➔*Gives the interpolation coefficients of conservative variable fields*

from volume averages to surface averages

For the Euler fluxes :

cell-centered stencils (in red, resp. green) for the sided reconstructions,

projection on one of the faces of this cell (surface integral of the polynomial)

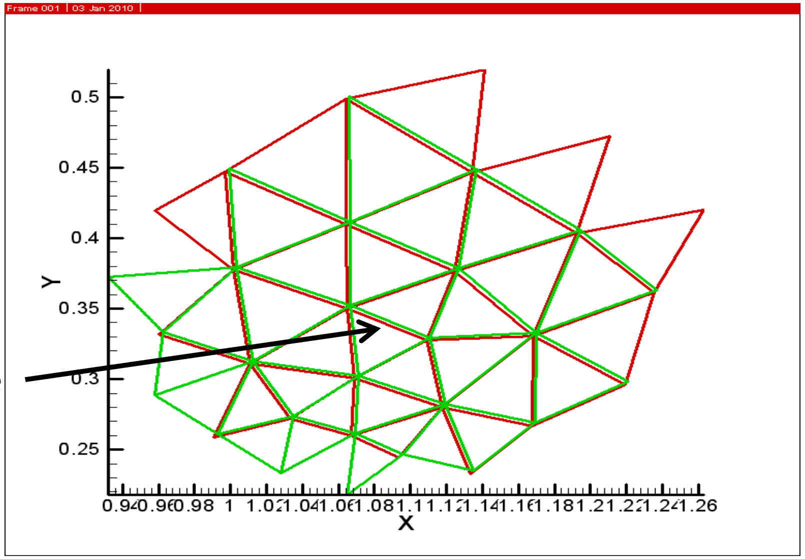
For the diffusive fluxes :

Interface-centered stencil (union red-green)

Projection of the gradients of the polynomial

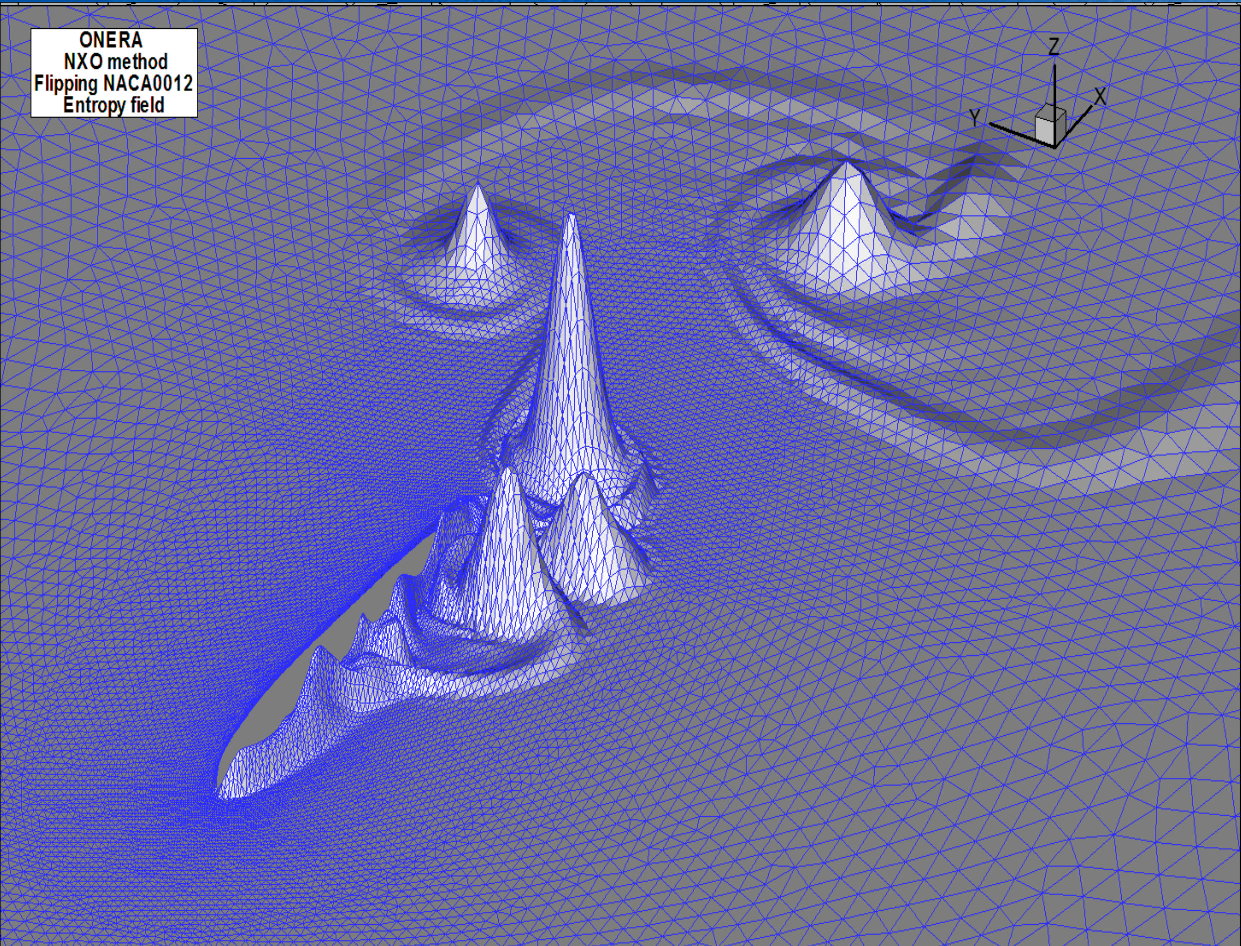
Same polynomial reconstruction used for the overset grid projection method

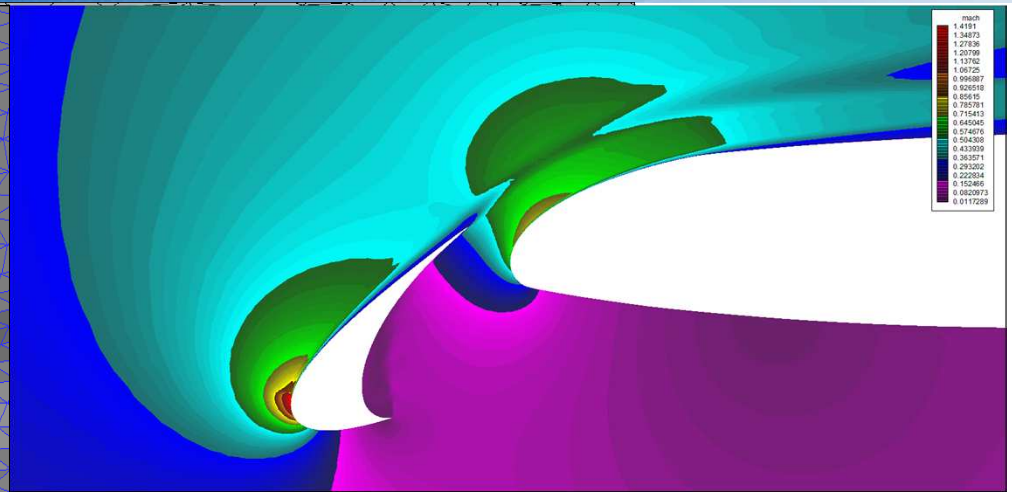
Target interface

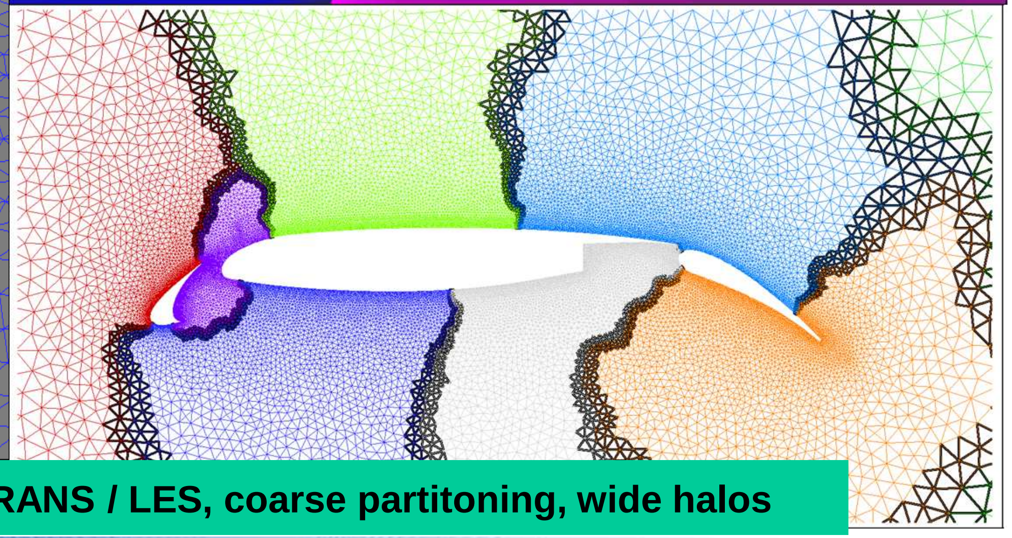


NextFlow : Spatially High-Order Finite Volume method for RANS / LES

Addressng the algorithmic efficiency barrier : accuracy versus number of dofs







NextFlow : Spatially High-Order Finite Volume method for RANS / LES, coarse partitoning, wide halos

Participation to the HO Workshop - Challenging the DG ?

Workshop 1

Laminar / DNS
Taylor-Green Vortex computed on a regular grid of tetraedra

Workshop 2

Unsteady Laminar : dual local time stepping
Heaving Naca12 in the unsteady wake of a cylinder,
computation of the frequency lock-in of the vortex shedding

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Euler
Isentropic vortex transport (right figure), results improved recently by using the flux reconstruction method

Workshop 3

Unsteay Laminar
Heaving and pitching naca12 at Reynolds numbers 1000 and 5000
demonstration of the grid convergence for both cases,
moderate computation costs

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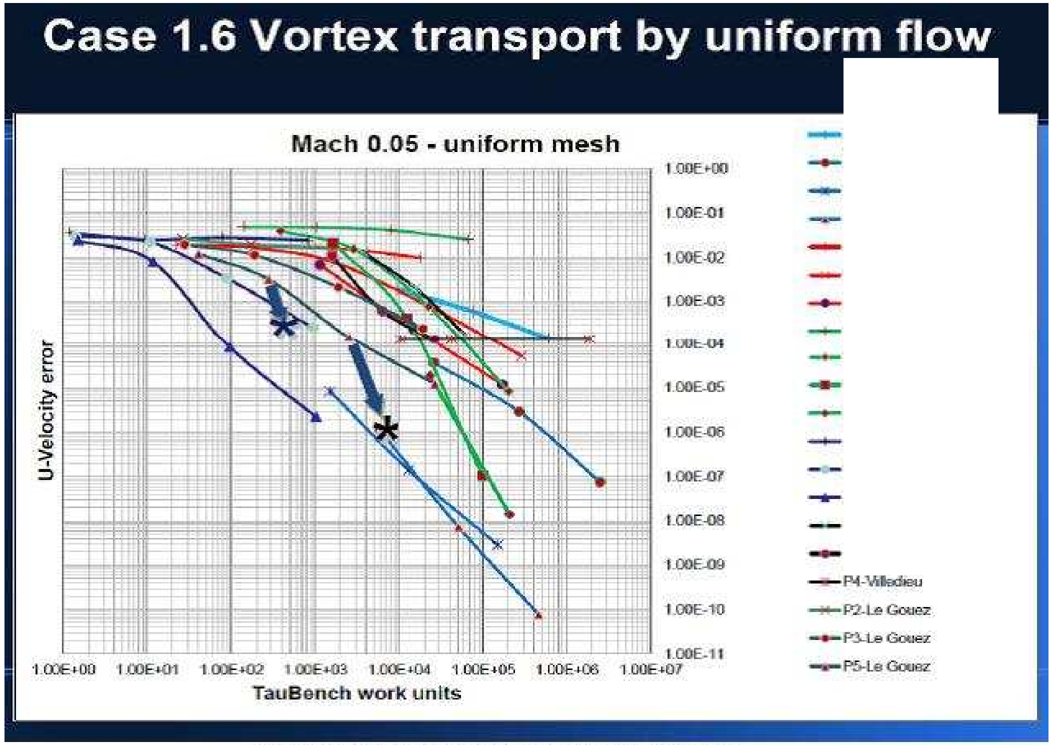
Euler, High order geometry
Ringleb Flow, with a novel formulation of the method on Finite Volumes of H.O. geometry. Convergence diffiulties on the finest grids, but good results in the coarser ones (even with low count of dof, accurate CR-BC on curved walls)

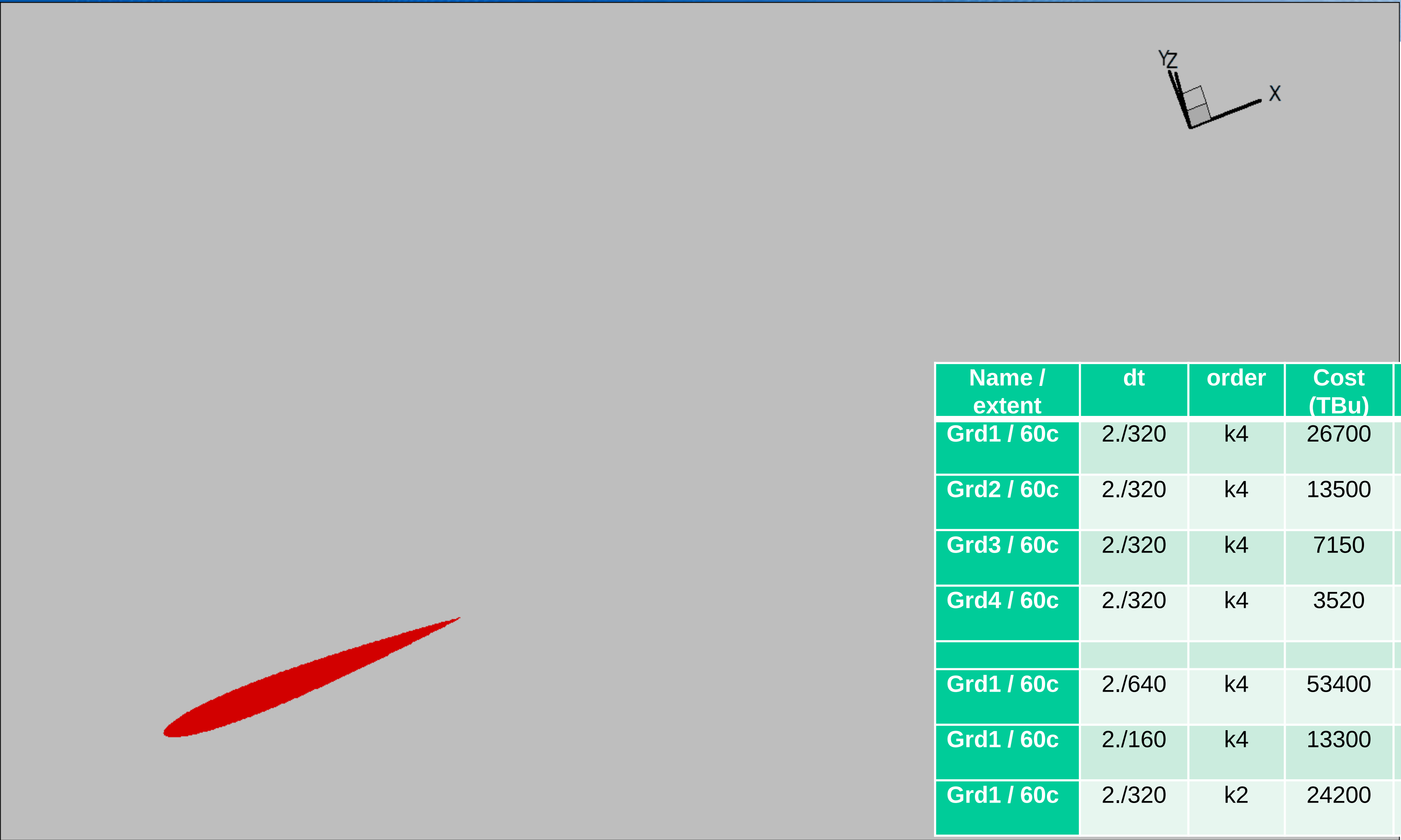
Workshop 3

Unsteay Laminar, BL3
Heaving and pitching Naca12, 3 different motions of the wing
Grid / time step / polynomial order convergence (to be confirmed),
moderate computation costs



From the Ref. 2, Convection of an isentropic vortex
Final error on the velocity components, Conformal Cartesian grids, Slow vortex, 50 Tc
Stars : centered scheme, red triangles : results obtained with the upwind scheme on grids M2 and M3,

HO Workshop 4 : Case BL3 Energy extracting

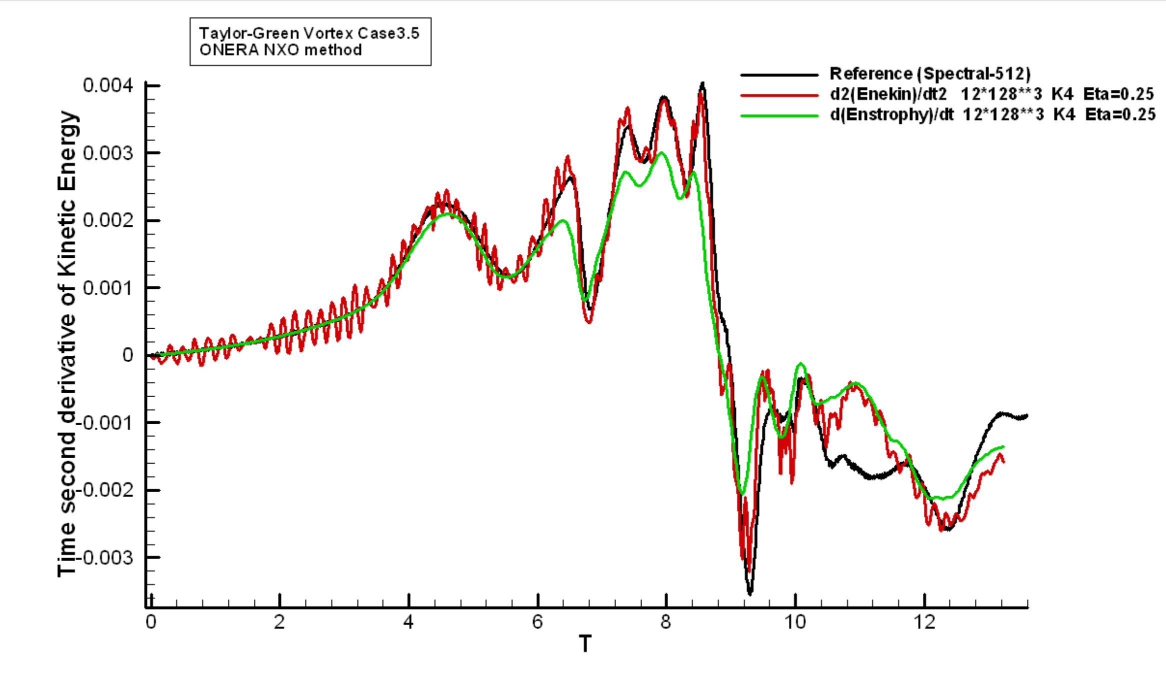


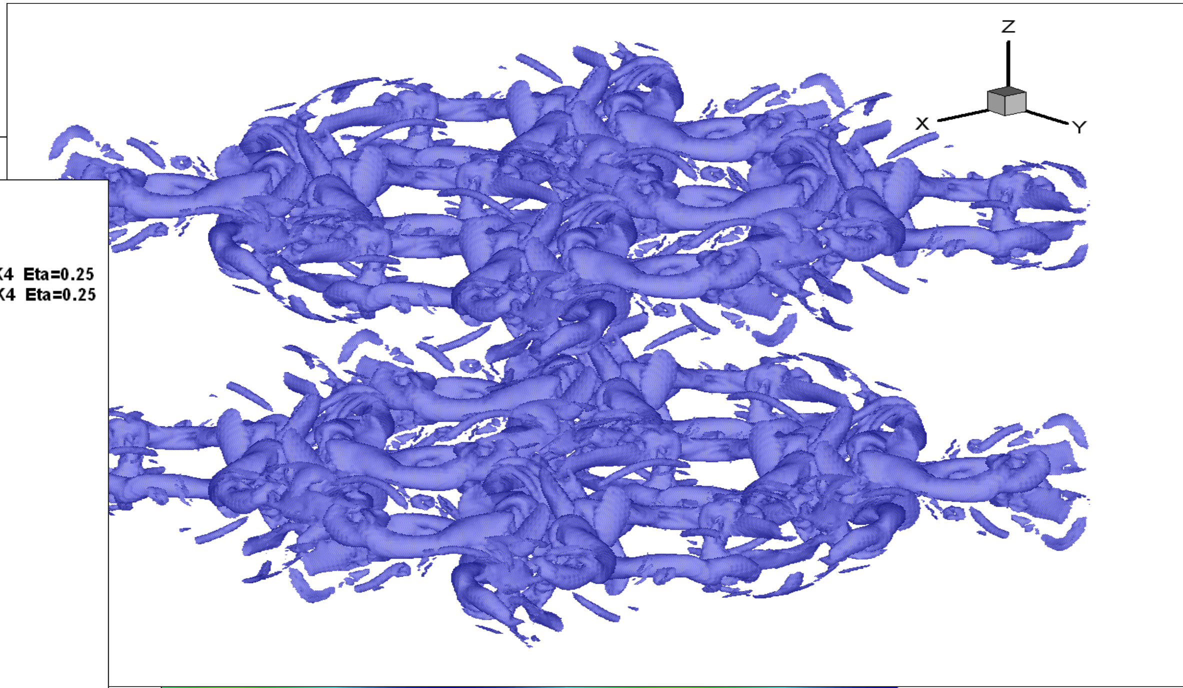
Name / extent	dt	order	Cost (TBu)	Ndof/ eqn	Y-Mometum	Work
Grd1 / 60c	2./320	k4	26700	59405	1.7076	0.3339
Grd2 / 60c	2./320	k4	13500	29972	1.7008	0.3365
Grd3 / 60c	2./320	k4	7150	15814	1.6820	0.3387
Grd4 / 60c	2./320	k4	3520	7916	1.6934	0.3548
Grd1 / 60c	2./640	k4	53400	59405	1.7099	0.3339
Grd1 / 60c	2./160	k4	13300	59405	1.7048	0.3342
Grd1 / 60c	2./320	k2	24200	59405	1.7131	0.3474

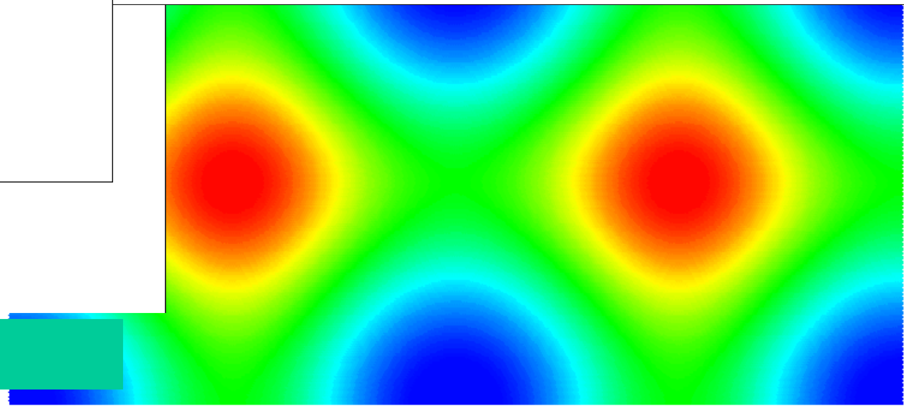
High Order CFD Workshop   Case 3.5 Taylor-Green Vortex

Comparison of time derivatives of enstrophy and scaled
Dissipation of kinetic energy

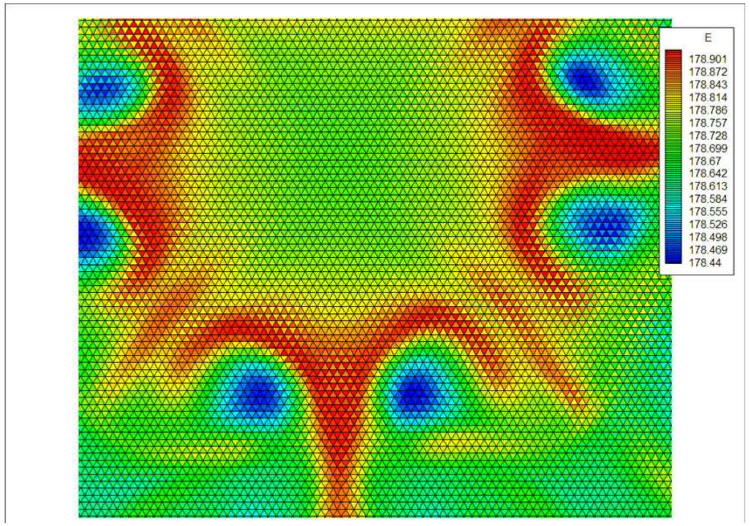
Occurrence of an acoustic phenomenon

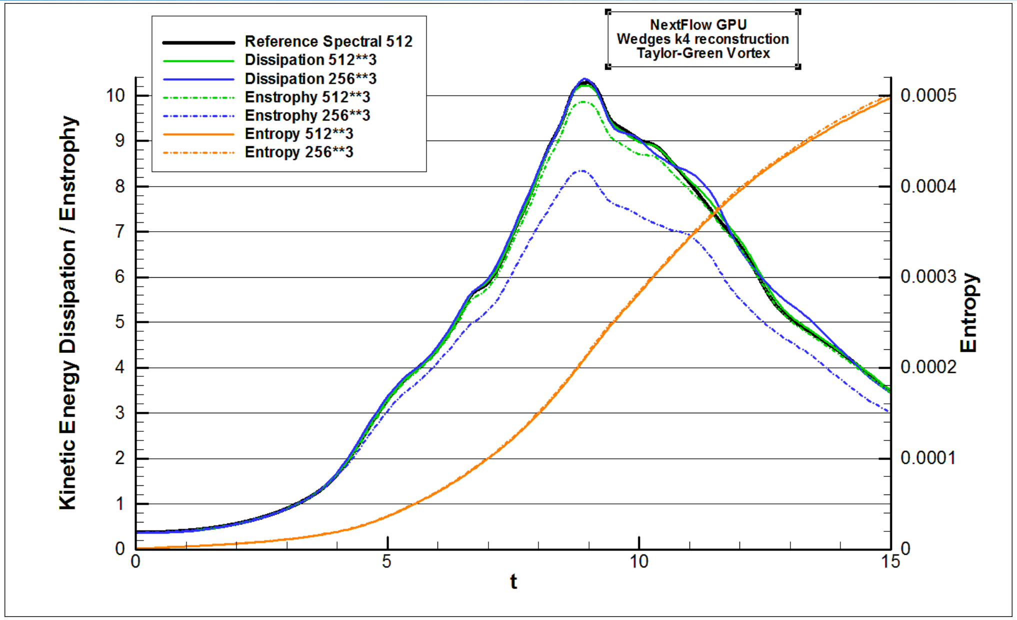






GPU implementation of the NextFlow solver





Taylor-Green Vortex Rey = 1600
Computations on wedges

Performance on each K20Xm GPU :

in k3 1,8e-8 s per RHS, 0.36s for 20 000 000 cells

in k4 2,5e-8 s per RHS, 0.50s for 20 000 000 cells

Taylor Green vortex 2563 - wall-clock =** **<u>12 hours on 16 IVY-Bridge processors (total 128 cores)</u> : 1600 hours CPU Intel core**

<u>25 minutes on 16 Tesla K20M GPU</u>

<u>By comparison, at the 1st HO CFD workshop , this case requested between 1100 and 33000 Intel core Cpu hours, depending on the numerical method</u>

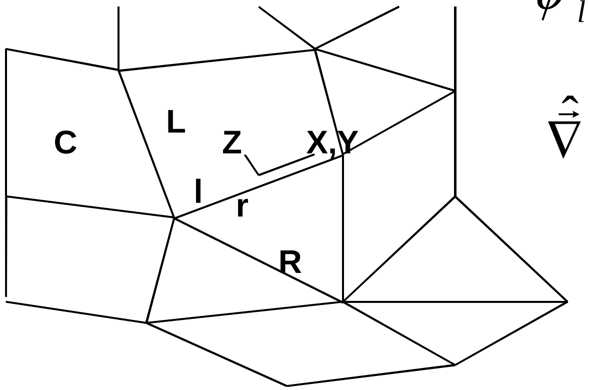
Taylor Green vortex 5123 - wall-clock :** ***<u>4 hours on 16 Tesla K20M GPUs</u>***

H.O. Full 3D Volume to face interpolation : Reconstruction and projection

$$\phi_a(X,Y,Z) = a_{\{ijk\}} X^i Y^j Z^k$$

$$\hat{\phi}_l = \sum_{c=1}^{ns} \lambda_c \overline{\phi}_c$$

$$\hat{\vec{\nabla}} \phi_l = \sum_{c=1}^{ns} \vec{\mu}_c \overline{\phi}_c$$



$$\psi = \sum_{s=1}^{ns} \varpi_s \left(\Omega_{c(s)} \overline{\phi}_{c(s)} - \int_{\Omega_{c(s)}} \phi_a(X,Y,Z)\, dV \right)^2$$

Reconstruction error functional

ns = Stencil size : nb of monomials + 50%

1/ Reconstruction in Left stencil centred on L

$$\frac{\partial \psi}{\partial a_{\{ijk\}}} = \sum_{s=1}^{ns} \varpi_s \left(-2\Omega_{c(s)} \Re^{ijk}_{c(s)} \overline{\phi}_{c(s)} + 2 {\Re^{ijk}_{c(s)}}^2 a_{\{ijk\}} + 2\Re^{ijk}_{c(s)} \sum_{\{ijk\} \neq \{i'j'k'\}} \Re^{i'j'k'}_{c(s)} a_{\{i'j'k'\}} \right) = 0$$

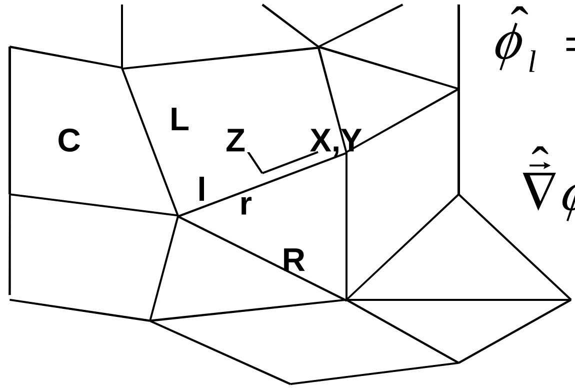
$$\Re^{ijk}_c = \int_{\Omega_c} X^i Y^j Z^k \, dV$$: Volume moment of order ijk

$$\mathrm{M}_{\{ijk\},c(s)} \overline{\phi}_{c(s)} + P_{\{ijk\},\{i'j'k'\}} a_{\{i'j'k'\}} = 0 \quad ==> \quad a_{\{ijk\}} = \kappa_{\{ijk\},c(s)} \overline{\phi}_{c(s)}$$

FV NXO method : Reconstruction and projection

$$M_{\{ijk\},c(s)}\overline{\phi}_{c(s)} + A_{\{ijk\},\{i'j'k'\}}\,a_{\{i'j'k'\}} = 0$$

$$a_{\{ijk\}} = \kappa_{\{ijk\},c(s)}\overline{\phi}_{c(s)}$$



$$\hat{\phi}_l = \sum_{c=1}^{ns}\lambda_c\overline{\phi}_c$$

$$\hat{\vec{\nabla}}\phi_l = \sum_{c=1}^{ns}\vec{\mu}_c\overline{\phi}_c$$

2/ Projection on the interface

$$\hat{\phi}_L = \frac{1}{S_{LR}}\int_{\partial\Omega_{LR}}\phi_a(X,Y,Z)\,dS = \frac{\int_{\partial\Omega_{LR}} X^iY^jZ^k\,dS}{S_{LR}}\,a_{\{ijk\}} = \nu_{\{ijk\}}a_{\{ijk\}}$$

$$\hat{\vec{\nabla}}\phi_L = \frac{1}{S_{LR}}\int_{\partial\Omega_{LR}}\vec{\nabla}\phi_a(X,Y,Z)\,dS = \left(i\,\nu_{\{i-1\,jk\}}\vec{e}_X + j\,\nu_{\{ij-1k\}}\vec{e}_Y + k\,\nu_{\{ijk-1\}}\vec{e}_Z\right)a_{\{ijk\}} = \vec{\eta}_{\{ijk\}}a_{\{ijk\}}$$

$$\hat{\phi}_L = \nu_{\{ijk\}}a_{\{ijk\}} = \nu_{\{ijk\}}\kappa_{g\{ijk\},c}\overline{\phi}_c = \lambda\,\overline{\phi}_c$$

$$\hat{\vec{\nabla}}\,\overline{\phi}_L = \vec{\eta}_{\{ijk\}}a_{\{ijk\}} = \vec{\eta}_{\{ijk\}}\kappa_{g\{ijk\},c}\overline{\phi}_{c(s)} = \vec{\mu}\,\overline{\phi}_c$$

FV NXO method : Inviscid fluxes options

$\overline{\phi}$ *: Volume average*

$\hat{\phi}$ *: surface average*

$\hat{\phi}\left(\overline{\phi}\right)$ *: NXO scheme*

$$\frac{\partial\left(\Omega \overline{W}\right)}{\partial t} + \sum_{n=1}^{ni} S_n \left(\hat{F}_i - \hat{F}_v\right)_n = 0$$

2 options for the inviscid fluxes
Characteristic Upwind or centred

$$\hat{F}_i = F_i(\widehat{W}_l(\overline{W}), \widehat{W}_r(\overline{W}))$$

Upwind scheme : one average flux evaluation from the left and right extrapolated average conservative variables, characteristic splitting 'state upwind'

$$\hat{F}_i = \widehat{F}_{i\,l}(\overline{F}) - \tau\theta_{\max}\left(\widehat{W}_l(\overline{W}) - \widehat{W}_r(\overline{W})\right)$$

Centred scheme : interpolation of the cell-average flux density tensor in all cells of the stencil to the interface
+ a stabilization term

Main inacuracy sources :

$$\hat{F} = F(\hat{W})$$

Upwind scheme

$$\overline{F} = F(\overline{W})$$

Centred scheme