McGill University

4^{th} High-order CFD Workshop

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Outline

1. High-order CFD Code
   - Code description
2. Code verification
   - NS solver verification via MMS
   - Example of solution verification
3. BS1 - DNS of the Taylor-Green Vortex

- **Governing equations**: Compressible Navier-Stokes equations
- **Discretization scheme**: high-order correction procedure via flux reconstruction (CPR)
- **Numerical Flux**: Roe scheme for inviscid terms and BR2 for viscous terms
- **Divergence computation method**: Chain rule (CR) for inviscid fluxes and the Lagrange polynomials (LP) for viscous fluxes
- **Solution method**: Backward Euler $\rightarrow$ Full Newton
- **Nodes**: GLL
- **Parallelization**: Open-MPI

Recent Developments (Farshad Navah)

- **Governing equations**: Compressible Reynolds-averaged Navier-Stokes equations closed by the *negative* Spalart-Allmaras (SA) turbulence model (ICCFD7-1902)
- **Solution method**: 15-digits accurate analytical Jacobian of RANS-SA(pos/neg), verified via complex step
- **Code verification**: Method of manufactured solutions (Euler, NS, RANS-SA)

Code verification in CFD

Simulation Step		**Sources of Error in CFD**
Reality		
$\Downarrow$	$\Longrightarrow$	Modelling error
Conceptual model		
$\Downarrow$	$\Longrightarrow$	Programming & Discretization errors
Numerical model		
$\Downarrow$	$\Longrightarrow$	Round-off & Iterative convergence errors
Numerical solution		

V&V		Sources of Error in CFD
Model validation	$\Longleftrightarrow$	Modelling error
$\Uparrow$		
Solution verification	$\Longleftrightarrow$	Discretization error
$\Uparrow$		
Code verification	$\Longleftrightarrow$	Programming errors
$\Uparrow$		
Solution process	$\Longleftrightarrow$	Round-off & Iterative convergence errors

Methods of Code Verification in CFD

- Method of analytical solutions
  Pros: -non-intrusive
  Cons: -limited range of models (no RANS solutions)
  -(often) over-simplified flows (ex: Couette flow)

- Method of manufactured solutions (MMS)
  Pros: -Covers all possible models/flow regimes
  -Verifies targeted boundary conditions
  -Allows for debugging
  Cons: -Creation of a proper MS is delicate wrt to model validity/numerical stability, etc.
  -Deployment needs expertise

Examples of Code and Solution Verification

Focus: Discretization and Programming Errors

- Round-off error
- Iterative convergence error

} $\longrightarrow$ Residual norm is at least 3 orders of magnitude lower than error norm

Manufactured solution:

$$\rho_{MS} = \rho_0 + \rho_x\, sin(a_{\rho x}\pi x/L) + \rho_y\, cos(a_{\rho y}\pi y/L) + \rho_{xy}\, cos(a_{\rho xy}\pi x/L)\, cos(a_{\rho xy}\pi y/L)$$

$$U_{MS} = u_0 + u_x\, sin(a_{ux}\pi x/L) + u_y\, cos(a_{uy}\pi y/L) + u_{xy}\, cos(a_{uxy}\pi x/L)\, cos(a_{uxy}\pi y/L)$$

$$V_{MS} = v_0 + v_x\, cos(a_{vx}\pi x/L) + v_y\, sin(a_{vy}\pi y/L) + v_{xy}\, cos(a_{vxy}\pi x/L)\, cos(a_{vxy}\pi y/L)$$

$$P_{MS} = p_0 + p_x\, cos(a_{px}\pi x/L) + p_y\, sin(a_{py}\pi y/L) + p_{xy}\, cos(a_{pxy}\pi x/L)\, cos(a_{pxy}\pi y/L)$$

$$E_{MS} = P_{MS}/((\gamma-1)\rho_{MS}) + \frac{1}{2}(U_{MS}^2+V_{MS}^2)$$

Domain:

$$\Omega = [0,1]^2$$

Grids:

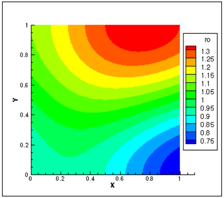
Series of doubling isotropic quads

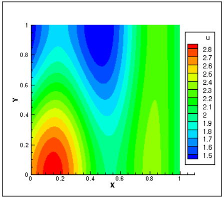
Viscous - (Full NS)

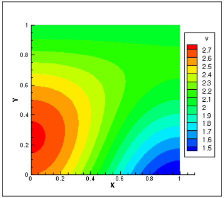
Solution: subsonic

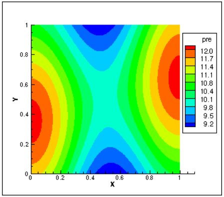
Viscosity: $\mu = 0.001$

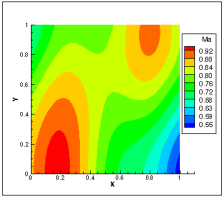
Boundary conditions: weak Dirichlet



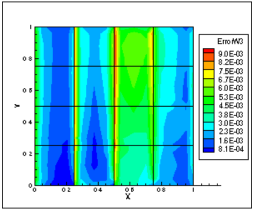




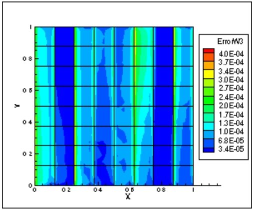




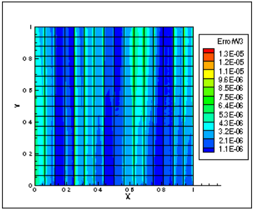
ρE error distribution versus grid refinement for $P4$



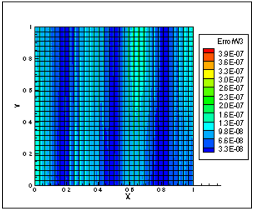
(a) 4 × 4



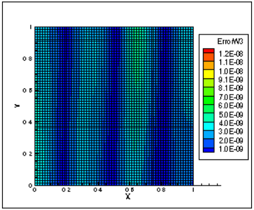
(b) 8 × 8



(c) 16 × 16



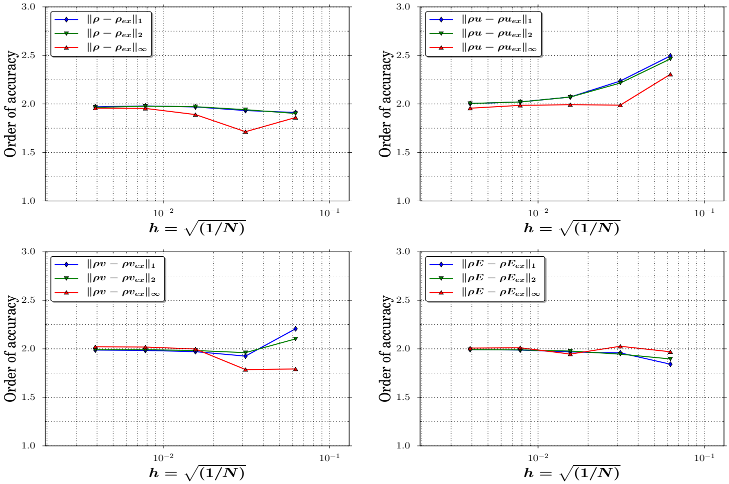
(d) 32 × 32



(e) 64 × 64

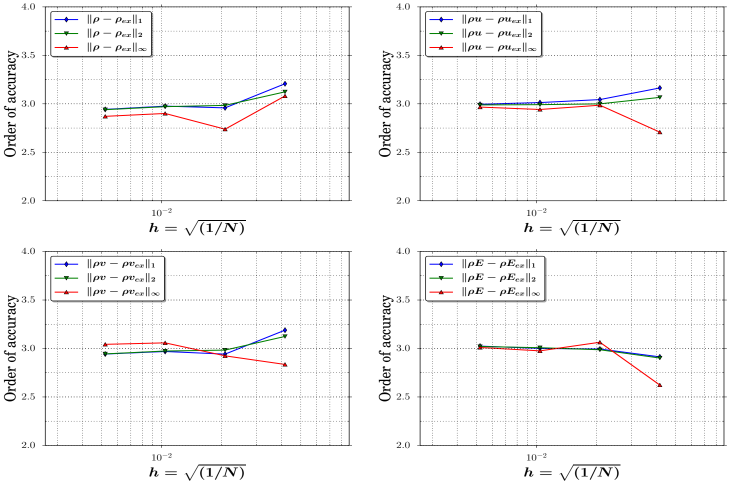
NS solver verification

Order of accuracy - P1



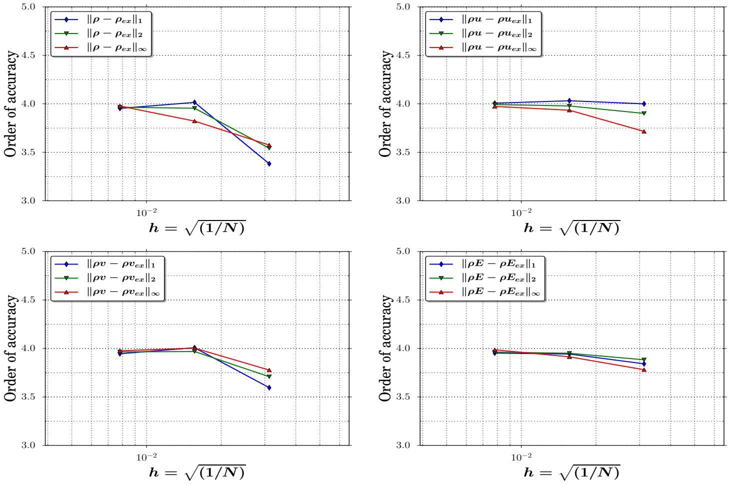
NS solver verification

Order of accuracy - **P2**



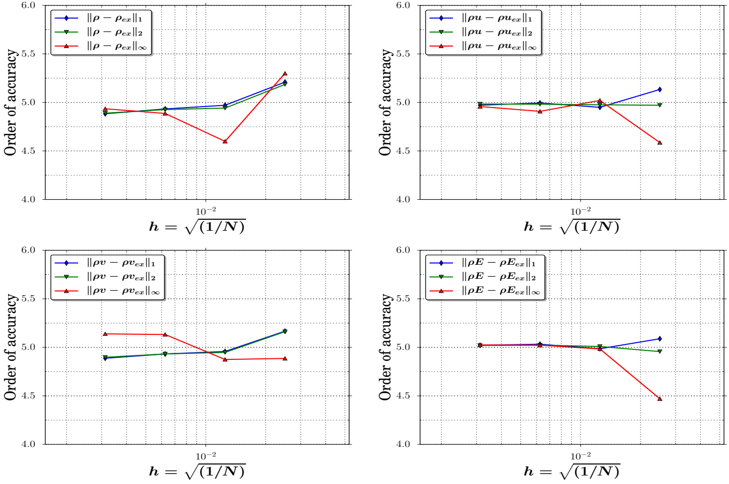
NS solver verification

Order of accuracy - $\mathbf{P3}$



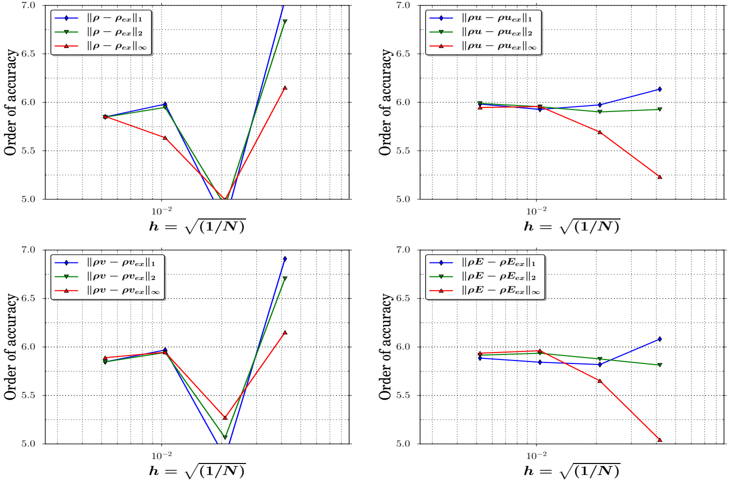
NS solver verification

Order of accuracy - $\mathbf{P4}$



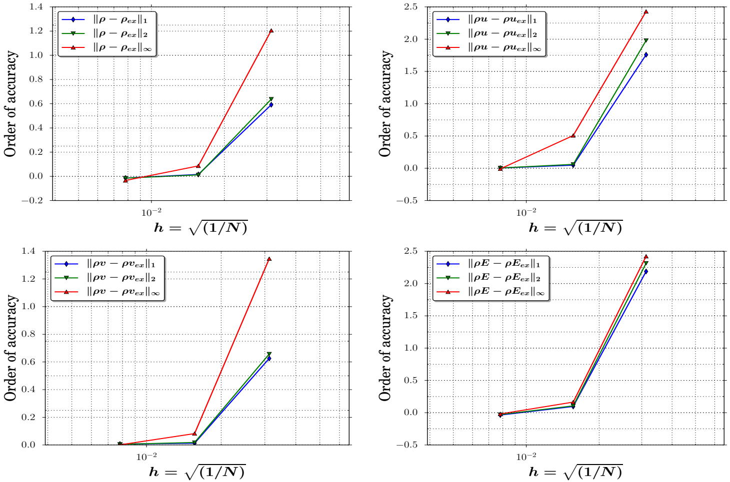
NS solver verification

Order of accuracy - $\mathbf{P5}$



NS solver verification

Order of accuracy - **P3** $q_x \rightarrow \mathbf{2.0} \times q_x$



Example of solution verification

Turbulent Boundary Layer from TMR

2D zero-pressure-gradient flat plate with $Re = 5 \times 10^6$, $Ma = 0.2$, $\chi_\infty = 0.3$ and $\chi_w = 0$:

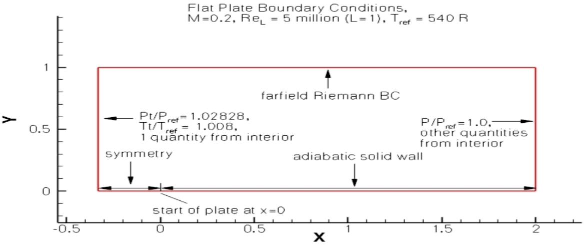
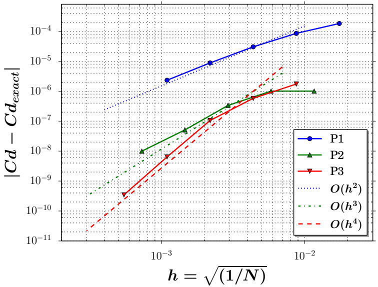


Figure: Domain and boundary conditions description

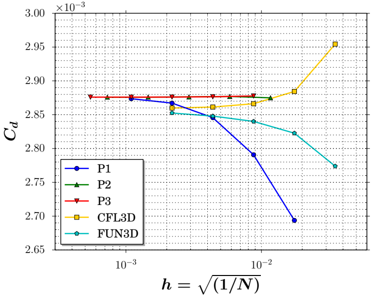
Discretization:

5 levels of h refinement

3 levels of p refinement: $P1$, $P2$ and $P3$



(a) Estimated C_d Error



(b) C_d

BS1 - DNS of Taylor-Green Vortex

Code optimization

CPR on Tensor-products $\longrightarrow$ Very sparse $\mathcal{D}$ and $\mathcal{L}$ operators

BR2 on Tensor-products $\longrightarrow$ Interior Penalty.

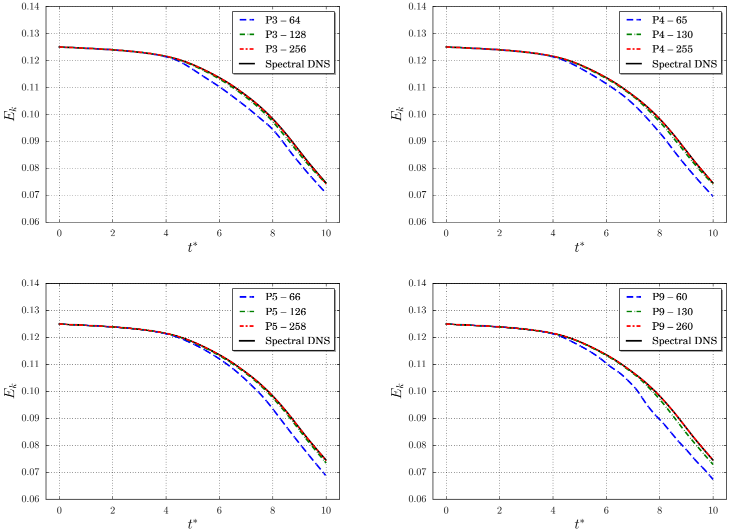
TGV for $P3-64$ is 5 times cheaper after optimization

12 simulations

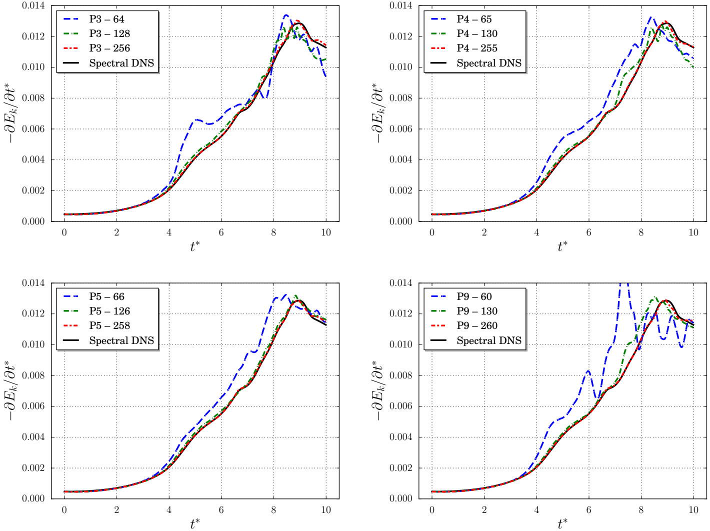
Resolution: 64^3, 128^3, 256^3 (based on dofs)

Polynomial: $P3$, $P4$, $P5$, $P9$

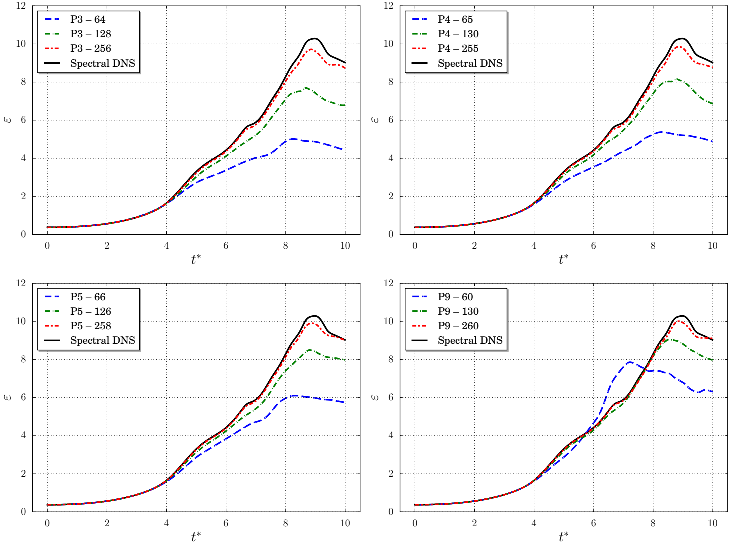
Kinetic Energy, E_k, vs t^*



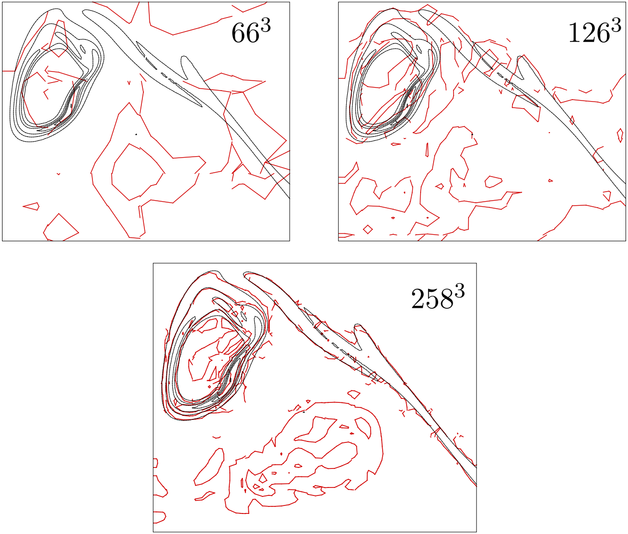
Kinetic energy dissipation, $-\partial E_k/\partial t$, vs t^*



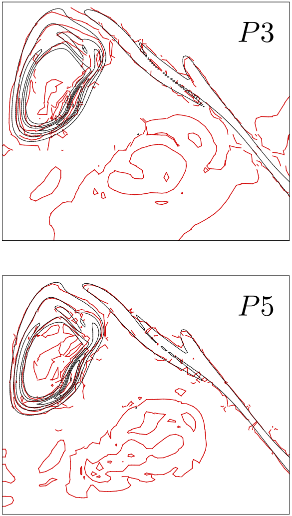
Enstrophy, ϵ, vs time, t^*



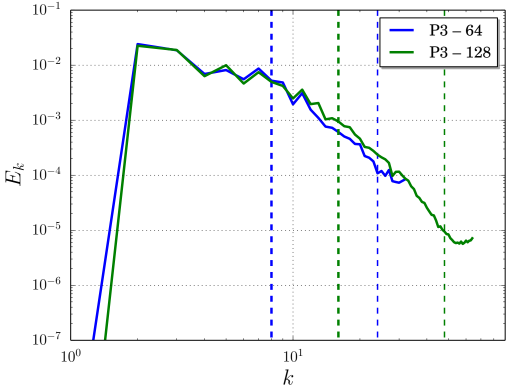
Vorticity isocontours at $x/L_0 = -\pi$ and $t^* = 8$ P = $\boldsymbol{P5}$



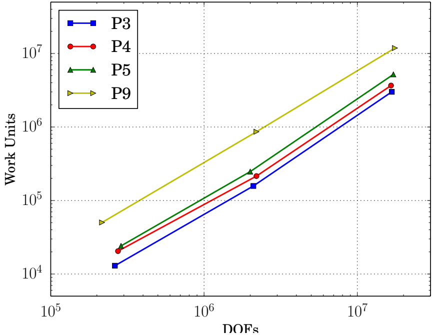
Vorticity isocontours at $x/L_0 = -\pi$ and $t^* = 8$ Res = $\mathbf{256^3}$



Energy spectrum



Work units vs DOFs



Thank you for your attention!

Questions?