Aghora : A high-order DG solver for turbulent flow simulations

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4^{th} International Workshop on High-Order CFD Methods
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Presentation outline

- Overview of Onera's CFD activities
- Aghora : a high-order DG solver for compressible flow
    - Current status of the solver
    - Turbulent flow simulations using Aghora (RANS, DNS, LES)
    - hp-adaptive simulations using Aghora
- The new prototype MICA : HO DG Multiscale InCompressible Adaptive solver
    - Current status of the MICA solver
    - DNS of the Taylor-Green Vortex (TGV)
    - Preliminary results from wavelet-based adaptive simulations
- On-going work and main perspectives

Overview of ONERA's industrial CFD software

Onera provides the aerospace industry with technical expertise and innovative CFD software:

- Aircrafts and helicopters : Airbus
- Turboengines : Safran
- Water turbines : EDF

Overview of ONERA's CFD activities:

- Development of hybrid/composite structured and unstructured FV methods: elsA for aerodynamics, Cedre for aerothermal applications.
- Development of a high-order CFD demonstrator based on compact variational methods: Aghora

The Aghora high-order solver: motivation

- Develop the next generation of industrial software : European projects TILDA (HO methods for LES using HPC) and SSeMID (Stability and Sensitivity Methods for Industrial Design, ITN), French project ELCI (coordinated by BULL) on pre/co/post-processing of HO solutions.
- Enhance the capacity of CFD solvers for tackling complexity (physics, geometry): very low dissipation and dispersion errors, accurate representation of curved boundaries.
- Designed for modern HPC architectures (multi-core, accelerators, etc.) : parallel efficiency thanks to compact schemes.

The Aghora code: current status

- The full set of compressible Navier-Stokes equations can be solved in 3D.
- Modal approach: the degrees of freedom (DoFs) are the coefficients of the polynomial expansion ($\mathbf{U}_j^k$) $\rightarrow$ natural implementation of multiscale turbulence models and p-adaptation.

$$\mathbf{u}_h(\mathbf{x}, t) = \sum_{k=1}^{N_p} \phi^k(\mathbf{x})\mathbf{U}_j^k$$

- Nodal approach (Raphaël Blanchard, PhD 2013-2016): Efficient evaluation of integrals at collocation points $\rightarrow$ validated on DNS of the TGV & RANS.
- Multi-element meshes (hexahedra, tetrahedra, prisms).
  - Jacobi polynomials for straight-sided elements (parallelepipeds, tetrahedra).
  - Modified Gram-Schmidt orthogonalization for general-shaped elements.
- Local Lax-Friedrichs, Roe, etc. numerical fluxes to approximate the convective fluxes across the interfaces.
- Discretization of viscous fluxes by BR2[1] scheme or Symmetric Interior Penalty method, SIP[2].

[1] F. Bassi & S. Rebay, J. Comput. Phys. (1997)

[2] R. Hartmann & P. Houston, J. Comput. Phys. (2008)

The Aghora code: current status

- Explicit time integration : up to 4th-order Runge-Kutta schemes
- Implicit time integration : several time-integration strategies[3] (solution of linear system by GMRES+left pre-conditioning (mixed precision algorithms), ESDIRK.
- Economical quadrature rules[4] (use element symmetries to reduce number of integration points)
- Shock capturing methods (Hartmann & Houston 2002, Hartmann 2013, Guermond et al. 2011)
- General thermodynamics : $p = p(\rho, \rho e)$ (e.g. stiffened gas, wan der Waals EOS)
- Turbulence modelling:
    - RANS Wilcox $\kappa - \omega$ model (Wilcox 1993): $\kappa - \omega$ and $\kappa - log(\omega)$ formulations (Bassi et. al 2005, 2009, Hartmann 2012). Spalart-Allmaras model.
    - DNS and LES simulations : Standard LES models (Smagorinsky, WALE), Variational Multiscale Simulation, VMS
    - Turbulent injection conditions: Synthetic Eddy Method (SEM)

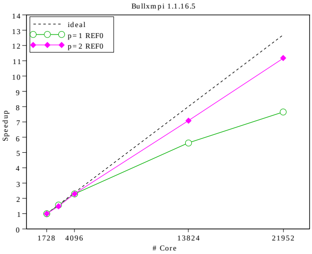
[3] F. Renac et al., J. Comput. Phys. (2013).

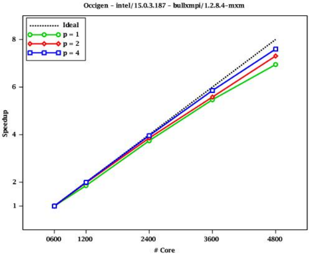
[4] P. Solin, K. Segeth, & I. Dolezel. High-order finite element methods. Chapman and Hall / CRC Press, 2003.

Strong scalability tests

Numerical experiments using the Taylor-Green vortex at $Re = 1\,600$
(96^3 elements, DG-P1 to DG-P4, SSP RK4)

- Sensitivity of the polynomial degree p on the speedup with REF0 version on *Curie*
  - Strong scalability analysis on a mesh with 336 elements
    - Ratio elts/ghosts per domain $\sim$ 4 at the largest scale
  - Receive-send message frequency (measured with 13 824 cores)
    - $p = 1$, $\sim$ 41 messages/s
    - $p = 2$, $\sim$ 15 messages/s
    - $p = 4$, $\sim$ 02 messages/s
  - On 21 952 cores with $p = 2$, efficiency of the speedup $\sim$ 88%





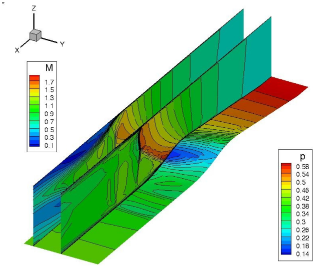
Strong scalability analysis

RANS using Spalart-Allmaras model with shock capturing

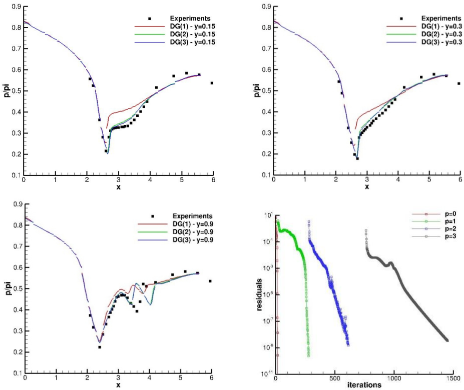
Shock/boundary layer interaction in a 3D transonic channel (F. Renac):

$Re = 1.7 \times 10^6,\ p_{out}/p_{i,0} = 0.64$

RANS/SA DG-P1 to DG-P3 (0.3 to 1.5 MDOFs)



Iso-contours of Mach number and wall pressure.

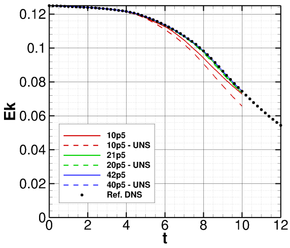


Decaying turbulence in a periodic box

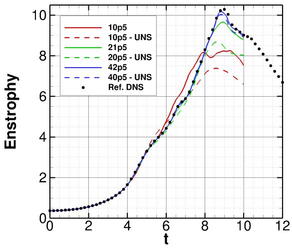
Mathieu Lorteau (post-doc)

Ref. Resolution	Simulation	# Elts.	Order	# Mdofs	Δt	CPU time (kWU)
64^3	STR-10p5	1 000	6	0.22	$1.9\,10^{-3}$	8.2
64^3	UNS-10p5	4 084	6	0.23	$1.1\,10^{-3}$	9.3
128^3	STR-21p5	9 261	6	2.00	$8.9\,10^{-4}$	125.5
128^3	UNS-20p5	31 225	6	1.75	$4.2\,10^{-4}$	175.7
256^3	STR-42p5	74 088	6	16.0	$4.4\,10^{-4}$	1877.8
256^3	UNS-40p5	229 004	6	12.8	$1.8\,10^{-4}$	2944.8

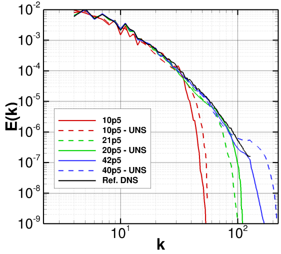
Turbulent kinetic energy



Enstrophy



Energy spectra at peak



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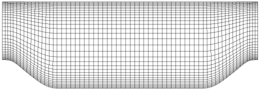
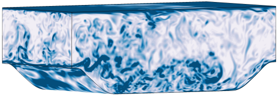
DNS and LES computations using Aghora: 2D periodic hill

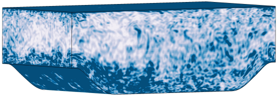
2D periodic hill at $Re_b = 2800$ up to $37\,000$: DNS & LES based on WALE approach
DG-P3 to DG-P5 on two levels of mesh refinement (0.5 to 4.2 MDOFs)



Fine 4th-order mesh $64 \times 32 \times 32$



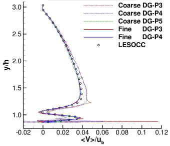
Vorticity magnitude $Re_b = 2\,800$

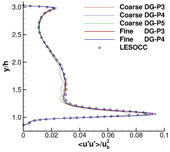


Vorticity magnitude $Re_b = 10\,595$

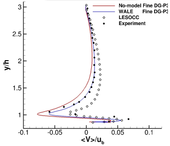
Profiles of mean vertical velocity $\langle V \rangle$ and Reynolds stresses $\langle u'u' \rangle$ at station x=0.5h

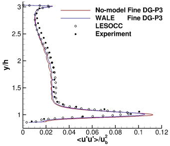
$Re_b = 2\,800$



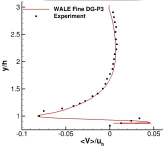


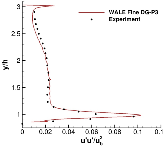
$Re_b = 10\,595$



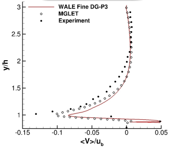


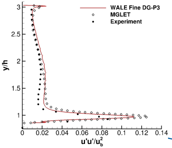
$Re_b = 19\,000$





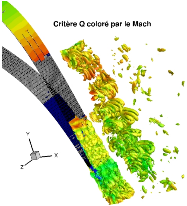
$Re_b = 37\,000$



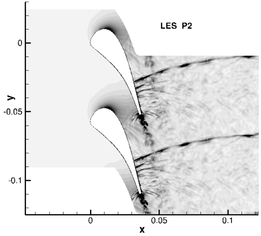


3D VKI LS89 LPT (Odile Labbé)

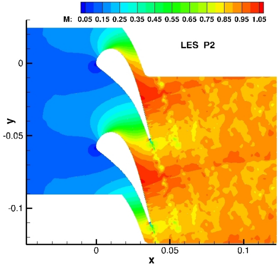
$Re_c = 1.25\,10^6$, under-resolved DNS DG-p2 (7.7 Mdofs)



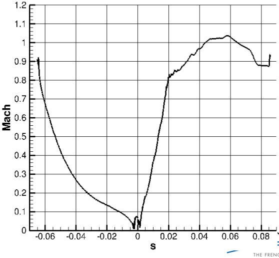
iso-surfaces of the Q-criterion coloured by Mach number



$\nabla \rho / \rho$



Iso-contours of Mach number, DG-P2 simulation

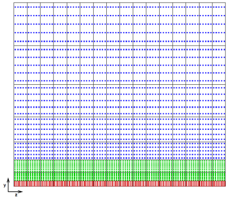


Mach number distribution around blade

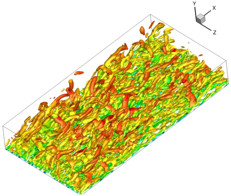
DNS computations using Aghora : hp-adapted channel at $Re_\tau = 590$

Jean-Baptiste Chapelier (PhD, 2010-2013)

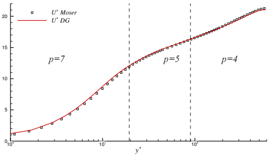
Numerical method	*# DOFs*	*# Integration points*	*order*
DG	7.02×10^5	1.12×10^6	$p = 4, 5, 7$
Spectral	1.28×10^8	4.32×10^8	-



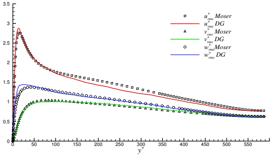
hp-adapted grid



Q-criterion coloured by streamwise velocity



Mean velocity profile

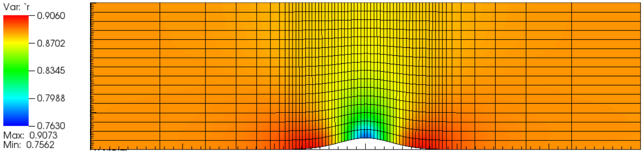


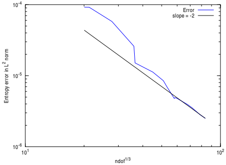
Fluctuating velocity profiles

Göktürk Kuru (Marie Curie PhD 2013-2016)

Adaptive simulation of the inviscid flow over a 2D bump

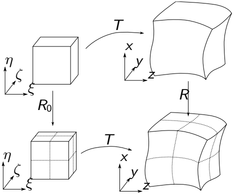
Refinement indicator → entropy error





Solve → Estimate → Mark → Refine

- Find the discrete solution
- Estimate the error
- Mark elements for refinement
- Refine marked elements



MICA : Multiscale InCompressible Adaptive DG solver

Brijesh Pinto (PhD, 2013-2016)

- High-order methods.
  - DG-SEM : Discontinuous Galerkin - Spectral Element Method.
  - Cartesian elements so far (methodology can easily be extended to HO elements).
  - Nodal/Lagrange basis functions at the element quadrature points : Gauss-Legendre-Lobatto (GLL) points.
  - Numerical fluxes used for inter-element coupling (centred/upwind schemes).
  - Parallel implementation.
- Robustness
  - Discontinuous velocity-pressure, stabilised discretization via penalty.
  - $P_N - P_N$ velocity-pressure space on non-staggered grids as opposed to $P_N - P_{N-2}$ on staggered grids.
  - Fractional step methods for pressure-velocity coupling (semi-explicit type procedure).
  - Time integration via 2nd-order implicit-explicit BDF2 scheme (IMEX-BDF2).
- Efficiency
  - Mesh adaptation via wavelet thresholding (*lazy* wavelet transform, WT)
  - Inexpensive and accurate error estimator, well suited to unsteady problems (DNS,LES)

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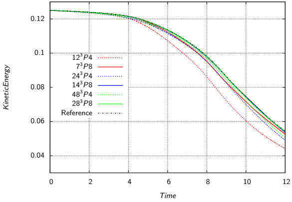
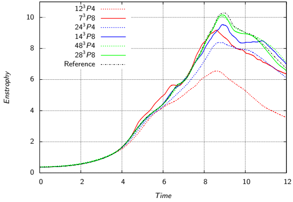
Brijesh Pinto (PhD, 2013-2016)

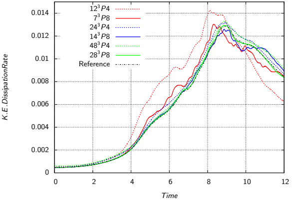
Ref. Resolution	Simulation	# Elts.	Order	# dofs
64^3	12p4	12^3	5	60^3
64^3	7p8	7^3	9	63^3
128^3	24p4	24^3	5	120^3
128^3	14p8	14^3	9	126^3
256^3	48p4	48^3	5	240^3
256^3	28p8	28^3	9	252^3



Evolution of kinetic energy



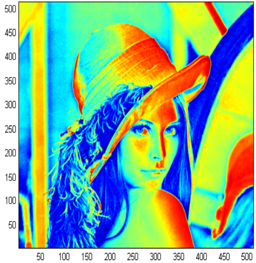
Evolution of enstrophy



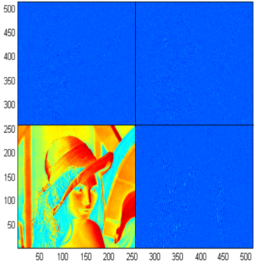
Evolution of energy dissipation.

Wavelet-based Multi-resolution analysis

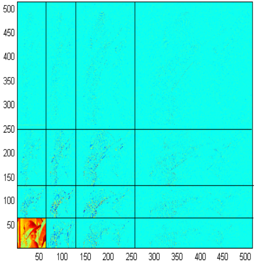
Brijesh Pinto (PhD, 2013-2016)



Physical space



1 level of the WT



2 levels of the WT

Wavelet-based Multi-resolution analysis (MRA)

- A sequence of spaces for approximation of finite energy functions.
- Non-linear approximation via thresholding.
- Consists of *scaling functions*(ϕ) $\rightarrow$ Basis for **coarse space** $\rightarrow$ Low pass filter
- Consists of *wavelets*(ψ) $\rightarrow$ Basis for **fine space** $\rightarrow$ Band pass filter

$$f(x) \simeq f^J_{\geq}(x) = \sum_k s_k^{J-m} \phi_k^{J-m}(x) + \sum_{j=J-m}^{J-1} \sum_{\substack{k=0 \\ |d_k^j| > \varepsilon}}^{2^j} d_k^j \psi_k^j(x) \qquad (1)$$

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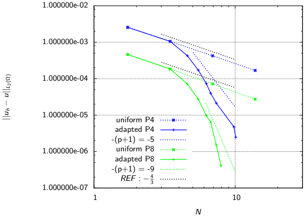
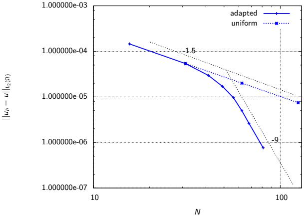
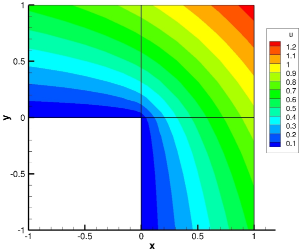
Examples for Adaptivity

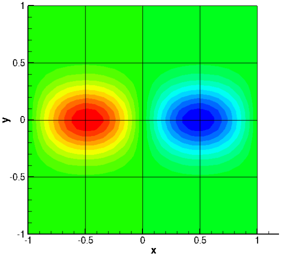
Brijesh Pinto (PhD, 2013-2016)







Scalar elliptic problem (DG-P4 & DG-P8)



Time-dependent Burgers equation (DG-P8)

- Aghora has been validated on a variety of configurations (RANS,DNS,LES)
- Major goal from 2016-onwards : ensure performance and robustness of the solver → compute flow configurations inscribed in Onera's and industrial application fields.
  - Ensure parallel efficiency of linear and non-linear solvers of systems arising from the DG discretization
  - Extend the capacity of the method to work in the all-Mach number domain (purely incompressible approach (MICA), compressible flow specific all-Mach number approaches)
- A great effort is being put in the implementation of performant hp-adaptation algorithms (dynamic load balancing, accurate refinement indicator) → PhD thesis on hp-adaptation for LES of pulsating flows (SSeMID ITN)
- Extend the capacity of the solver to perform simulations with multi-physics coupling (aeroacoustics, aeroelasticity, aerothermal)

ONERA participation to HOW4 test cases (Aghora)

- BL2 - Laminar shock/Boundary layer interaction
- BS1 - Taylor-Green Vortex at $Re = 1600$
- AR2 - RANS of the transonic turbulent flow in a 3D channel with a swept bump
- AS2 - Spanwise periodic DNS/LES of transitional turbine cascades (T106C)