BS1 - DNS of the transition of the Taylor-Green vortex, Re=1600

Introduction and result summary

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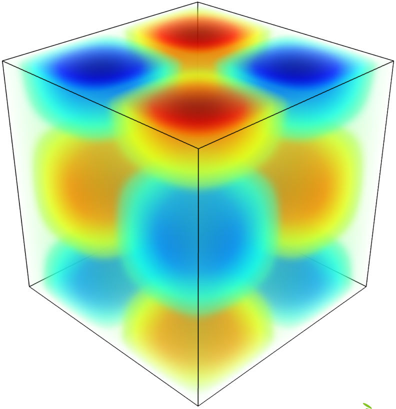
Doc. ref.:

Periodic domain

$$(x, y, z) \in \Omega = [-\pi L, \pi L]^3$$
$$(x', y', z') = (x, y, z)/L$$

At t=0

$$u = V_0 \sin x' \cos y' \cos z'$$
$$v = -V_0 \cos x' \sin y' \cos z'$$
$$w = 0$$
$$p = p_0 + \rho_0 V_0^2 \left(\cos 2x' + \cos 2y'\right)\left(\cos 2z' + 2\right)/16$$
$$T = T_0$$



with

$$Re = \rho_0 V_0 L/\mu = 1600$$
$$M = V_0/\sqrt{\gamma R T_0} = 0.1$$

van Rees *et al.*, JCP 2011

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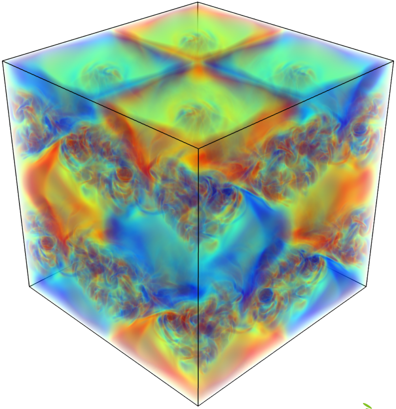

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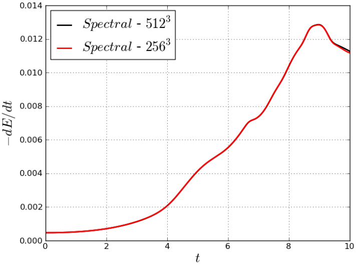

Spectral convergence ~ 256^3

Kinetic energy

$$E_k = \frac{1}{\rho_0} \int_\Omega \rho \frac{\mathbf{u} \cdot \mathbf{u}}{2} \, d\Omega \, ,$$

Dissipation rate and relation to enstrophy

$$\epsilon = -\frac{dE_k}{dt} = \underbrace{2\frac{\mu}{\rho_0} \int_\Omega \mathbf{S}^d : \mathbf{S}^d \, d\Omega}_{\epsilon_1 = 2\nu\mathcal{E}} + \underbrace{\frac{\mu_v}{\rho_0} \int_\Omega (\nabla \cdot \mathbf{v})^2 \, d\Omega}_{\epsilon_2 \approx 0} - \underbrace{\frac{1}{\rho_0} \int_\Omega p \, \nabla \cdot \mathbf{v} \, d\Omega}_{\epsilon_3 \approx 0}$$

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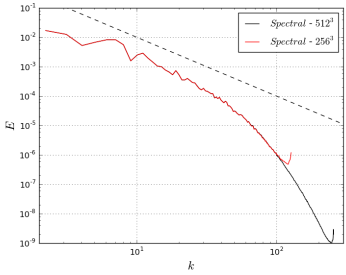
Testcase description
Reference : pseudo-spectral 512 modes (courtesy UCLouvain)



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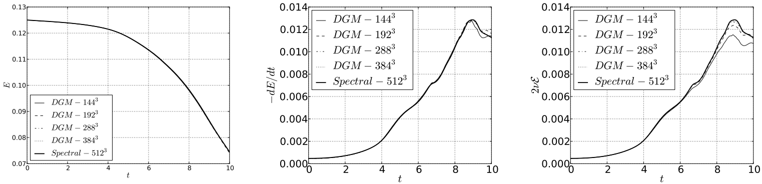

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Errors with respect to pseudo spectral solution

$$\Delta\epsilon_1 = \left\| \frac{dE_k}{dt} - \left(\frac{dEk}{dt}\right)^* \right\|_{t,\infty}$$

$$\Delta\epsilon_2 = \left\| 2\nu\mathcal{E} - 2\nu\mathcal{E}^* \right\|_{t,\infty}$$

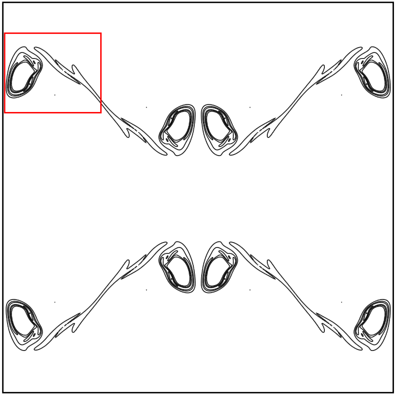
Numerical dissipation : difference with DNS

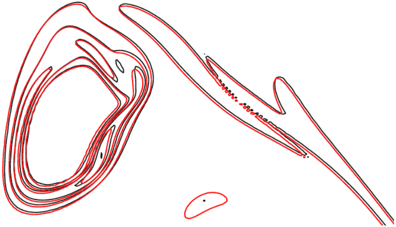
$$\Delta\epsilon_3 = \left\| \frac{dE_k}{dt} + 2\nu\mathcal{E} \right\|_{t,\infty}$$

with

$$\|a\|_{t,\infty} = \max_{t\in[0,10]} |a|$$

CPU time to reach $t_c = 10$, measured in WU





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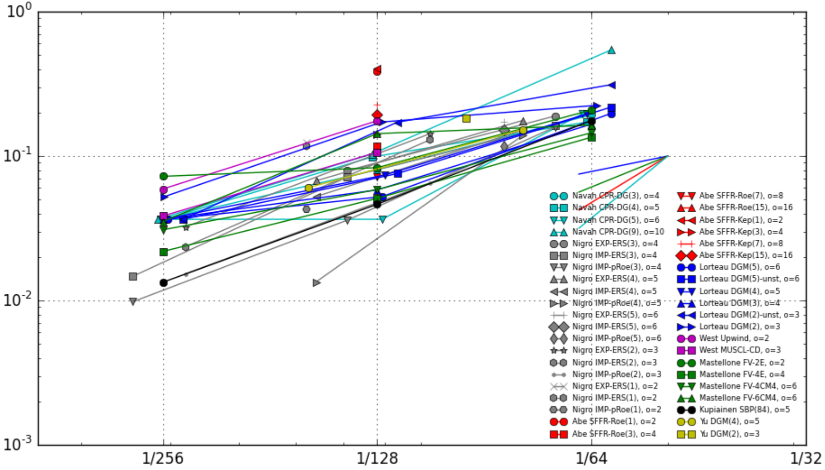
Studies performed on the case

- Y. Abe, I. Morinaka, T. Haga, T. Nonomura and K. Miyaji (JAXA) - order convergence studies (1,3,7 and 15) on 128 mesh using SFFR with Roe and kinetic energy preserving flux (+ studies on unstructured meshes) ;
- M. Kupiainen, J. Nordström and P. Eliasson (LIU/SHMI/FOI) - grid convergence studies (64/128/256) with SBP(84) scheme ;
- M. Lorteau, M. de la Llave Plata, V.Couaillier (Onera) - Aghora : grid convergence studies (64/128/256) with DGM at orders 2,3,4 and 5 ;
- A. Mastellone (CIRA) - SparkLES : grid convergence studies (64/128/256) FVM 2nd and 4th order explicit, 4 and 6th order compact schemes ;
- F. Navah and S. Nadarajah (McGill) : grid convergence studies (64/128/256) using CPR-DG(Roe/BR2) at interpolation orders 3,4,5 and 9 ;
- A. Nigro (UCalabria/UBergamo) grid convergence studies of DGM schemes Exp-ERS (explicit), Imp-ERS (Rosenbrock), Imp-pRoe (Rosenbrock, preconditioned Roe) at orders 1,2,3,4 and 5 ;
- A. West (CD-Adapco) : convergence studies (128/256) using FVM Upwind (2) and MUSCL-CD(3)
- J. Yu (BUAA) DGM studies 80/96/160 at orders 2,4 ;

Global results

Error on measured dissipation : resolution



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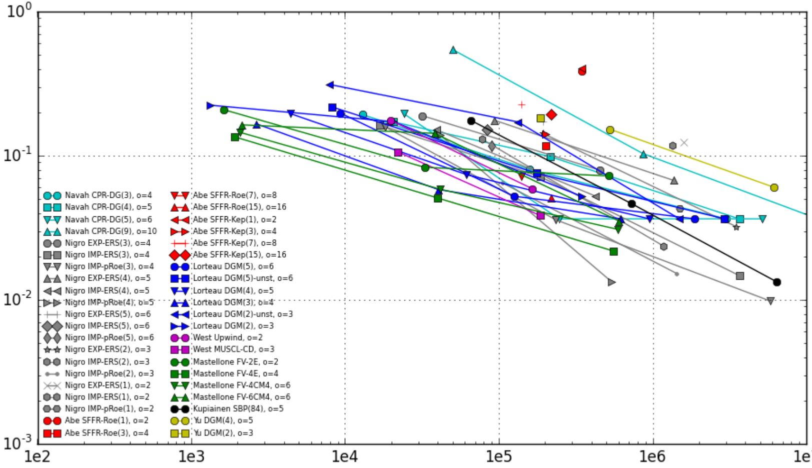
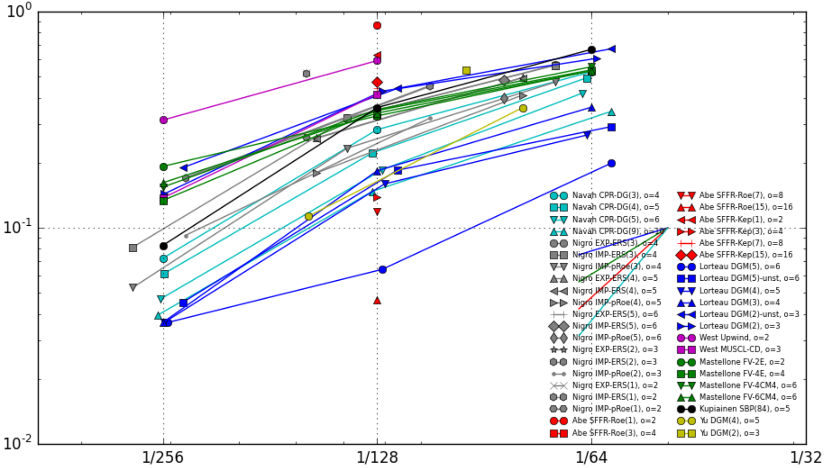
Error on measured dissipation : cpu time



Global results

Error on enstrophy dissipation : resolution



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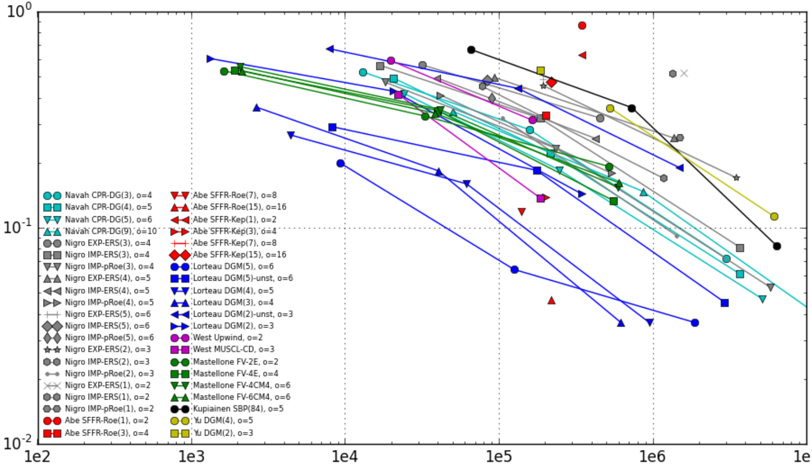
Global results

Error on enstrophy dissipation : cpu time



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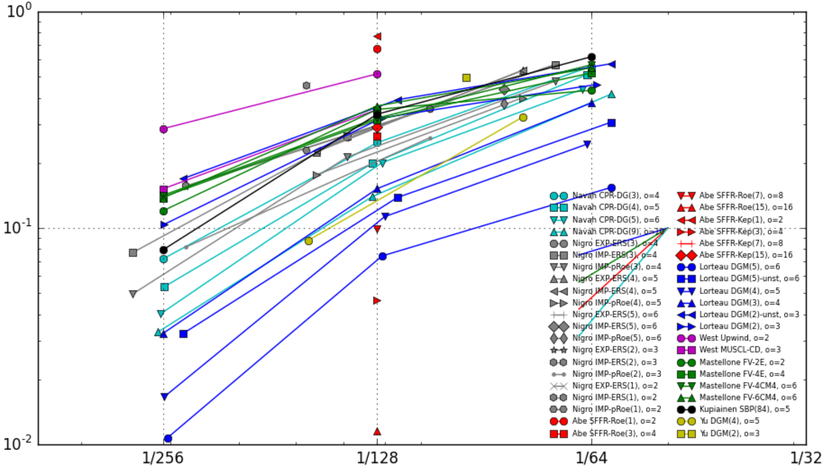
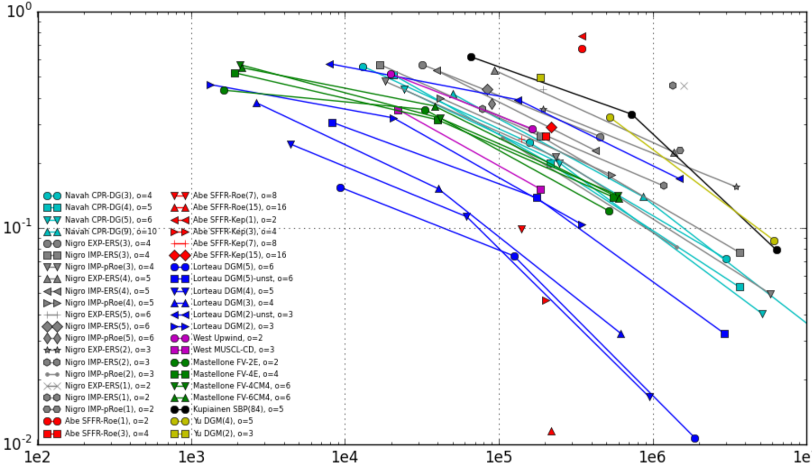


10^0
10^{-1}
10^{-2}
1/256
1/128
1/64
1/32
Navah CPR-DG(3), o=4
Navah CPR-DG(4), o=5
Navah CPR-DG(5), o=6
Navah CPR-DG(9)
Nigro EXP-ERS(3), o=4
Nigro IMP-ERS(3), o=4
Nigro IMP-pRoe(3), o=4
Nigro EXP-ERS(4), o=5
Nigro IMP-ERS(4), o=5
Nigro IMP-pRoe(4), o=5
Nigro EXP-ERS(5), o=6
Nigro IMP-ERS(5), o=6
Nigro IMP-pRoe(5), o=6
Nigro EXP-ERS(2), o=3
Nigro IMP-ERS(2), o=3
Nigro IMP-pRoe(2), o=3
Nigro EXP-ERS(1), o=2
Nigro IMP-ERS(1), o=2
Nigro IMP-pRoe(1), o=2
Abe SFFR-Roe(1), o=2
Abe SFFR-Roe(3), o=4
Abe SFFR-Roe(7), o=8
Abe SFFR-Roe(15), o=16
Abe SFFR-Kep(1), o=2
Abe SFFR-Kep(3), o=4
Abe SFFR-Kep(7), o=8
Abe SFFR-Kep(15), o=16
Lorteau DGM(5), o=6
Lorteau DGM(5)-unst, o=6
Lorteau DGM(4), o=5
Lorteau DGM(3), o=4
Lorteau DGM(2)-unst, o=3
Lorteau DGM(2), o=3
West Upwind, o=2
West MUSCL-CD, o=3
Mastellone FV-2E, o=2
Mastellone FV-4E, o=4
Mastellone FV-4CM4, o=6
Mastellone FV-6CM4, o=6
Kupiainen SBP(84), o=5
Yu DGM(4), o=5
Yu DGM(2), o=3

Global results

Difference between measured and theoretical dissipation : cpu time



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Conclusions

- whatever method, 256 seems necessary to perform DNS, otherwise LES
- order of accuracy not obtained
- higher order at isodof pays off
- error on measured dissipation always lower than error on enstrophy and closer by for all methods and less dependent on order. Natural tendency of numerics to compensate lack of resolution ?
- Kinetic energy preservation / central discretisation can be counterproductive, not only for measured dissipation but also for enstrophy ; require SGS.
  - Abe : comparison Kep/Roe
  - High order FV (Cira)
  - SBP(84)
- not much impact of order on FVM schemes ? ?
- similar, but surprisingly not too close errors for DGM ;
  - importance of flux function for DGM ?
  - difference structured/unstructured mesh quite pronounced ?
- difficult to measure efficiency. Eg. precision in dof compensated by speed for FV/Cira ~ structured operator
- difference measured / theoretical dissipation good measure for precision - order

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