BL3 — Heaving and Pitching Airfoil

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Overview

This problem is aimed at testing the accuracy and performance of high-order flow solvers for problems with deforming domains. A NACA 0012 airfoil is undergoing a smooth flapping-type motion, starting from rest at zero angle of attack and ending at a one chord length higher position at the end of the motion at time T. The metrics used to assess the accuracy of the solution are the total energy (i.e. integrated power) which the flow exerts on the airfoil, and the vertical impulse imparted by the flow on the airfoil (integrated vertical force). The viscosity is constant and the Reynolds number with respect to the chord length is Re = 1000.

Governing Equations

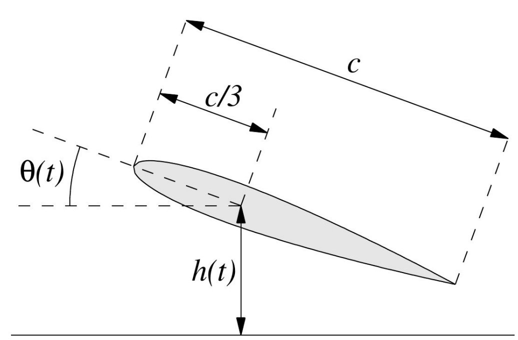
The governing equations for this problem are the 2D compressible Navier-Stokes equations with a constant ratio of specific heats equal to 1.4 and a Prandtl number of 0.72. Two boundary conditions are imposed: far-field characteristic conditions at the outer domain and no-slip adiabatic wall condition on the moving airfoil.

Geometry

The geometry consists of a NACA 0012 airfoil with chord length $c = 1$, with geometry modified to give zero trailing edge thickness:

$$y(x) = \pm 0.6\,(0.2969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1036x^4),\ x \subset [0, 1].$$

The far-field boundary should be located at least 100 chord-lengths away from the airfoil. The airfoil undergoes a smooth upward motion of one chord length for the duration of T=2 time units, by heaving and pitching about a point located at the airfoil 1/3 chord location (see figure). We consider three different motions, with different properties and difficulties. We first define the following polynomials:



$$b_1(t) = t^2(t^2 - 4t + 4)$$
$$b_2(t) = t^2(3 - t)/4$$
$$b_3(t) = t^3(-8t^3 + 51t^2 - 111t + 84)/16$$

In terms of these, we define the vertical displacement $h(t)$ and the pitching angle $\theta(t)$ for the three test cases according to below.

Case 1 (Pure heaving)	**Case 2** (Flow aligning)	**Case 3** (Energy extracting)
$h(t) = b_2(t)$ $\theta(t) = 0$	$h(t) = b_2(t)$ $\theta(t) = A_2 \cdot b_1(t)$	$h(t) = b_3(t)$ $\theta(t) = A_3 \cdot b_1(t)$

where the constants $A_2 = 60\pi/180$ and $A_3 = 80\pi/180$.

Flow Conditions

The free-stream has a Mach number $M_\infty = 0.2$ and is horizontal, so that θ is the airfoil angle of attack. The Reynolds number based on the chord of the airfoil is Re = 1000. The initial condition at time $t = 0$ is the steady-state solution for the initial position $h = 0$, $\theta = 0$. To simplify post-processing, we assume convenient units in which the airfoil chord is $c=1$ and the free-stream density and speed are unity, so that the free-stream conservative state vector is

$$[\rho, \rho u, \rho v, \rho E] = \left[1, 1, 0, 0.5 + 1/[M^2\gamma(\gamma - 1)]\right]$$

Output Quantities

The first output from the simulation is the work (energy) which the fluid exerts on the airfoil during the motion, which can be written as:

$$W = \int_0^T \boldsymbol{F}(t) \cdot \boldsymbol{v}_0 \, dt + \int_0^T \boldsymbol{T}(t) \cdot \boldsymbol{\omega} \, dt = \int_0^T F_y(t)\dot{h}(t)\, dt + \int_0^T T_z(t)\dot{\theta}(t)\, dt.$$

Here, $F(t) = (F_x(t), F_y(t))$ is the force imparted by the fluid on the airfoil, $T(t) = (0, 0, T_z(t))$ is the torque imparted by the fluid on the airfoil about the 1/3 chord pivot point, $v_0 = \dot{h}(t)$ is the velocity of the pivot point, and $\omega_0 = (0, 0, \dot{\theta})$ is the angular velocity of the airfoil about the pivot point. Note that this output can be equivalently computed as

$$W = \int_0^T \int_{\text{airfoil}} \vec{v}_G(t) \cdot \vec{f}_{\text{surf}}(t) \, ds \, dt,$$

where $v_G(t)$ is the velocity of the surface of the airfoil and $f_{surf}(t)$ is the surface stress vector.

The second output is the vertical impulse from the fluid onto the airfoil during the motion:

$$I = \int_0^T F_y(t)\, dt\,.$$

Requirements

- Perform the indicated simulation for the three test cases. Calculate the quantities W and I for each case, and perform a grid/timestep convergence study to get the values as accurate as possible. Record the work units.
- Provide the work units, the converged output values, nDOFs in the discretization, and the distance to the far-field boundary for each case. Submit this data to the workshop contact.