BI2 - Smooth bump, BL1 - Laminar airfoil & BR1 - Turbulent airfoil

Marshall C. Galbraith[1] and Carl Ollivier-Gooch[2]

[1]Aerospace Computational Design Laboratory
Massachusetts Institute of Technology

[2]Advanced Numerical Simulation Laboratory
The University of British Columbia

Jun 4, 2016

Motivation

Order of Accuracy Code Verification

- Is the code correct?
- Non-intrusive (unlike Method of Manufactured Solutions)
  - Can be applied to “off the shelf” codes
- Inviscid: Smooth Gaussian Bump
- Laminar: $Re = 1000$ Joukowski Airfoil
- RANS: $Re = 1e6$ Joukowski Airfoil

The Importance of Meshing

- Misnomer: “High-order methods are less sensitive to the mesh”
- Compact schemes are less sensitive to poor shape (e.g. misaligned anisotropy) and mesh irregularity
- Solution/Output accuracy is sensitive to global distribution

Contributions

- James Thomas and Kyle Anderson (NASA Langley, USA)
  - Finite Volume (Fun3D-FV and USM3D)
  - Continuous Galerkin (Fun3D-SUPG)
- Ryan Glasby and Taylor Erwin (U. of Tennessee, USA)
  - Continuous Galerkin (COFFE)
- Phillip Kirshen (MIT, USA) and Micheal List (Wright Patt AFB, USA)
  - Finite Volume (Star CCM)
- Arthur Huang (MIT, USA)
  - Finite Volume (Fluent)
- Carl Olivier Gooch and Gary Yan (U. of British Columbia, Canada)
  - Finite Volume (ANSLib)
- Krzysztof Fidkowski (U. of Michigan, USA)
  - Discontinuous Galerkin (xflow)

Contributions

- Marshall Galbraith (MIT, USA)
  - Finite Volume (OVERFLOW, CFD++, Fine Turbo, Fluent)
  - Discontinuous Galerkin (ProjectX)
- Shengye Wang, Yaming Chen, Xiaogang Deng (National U. of Defense Technology, China)
  - Weighted Compact Nonlinear Scheme Finite Difference (WCNS)
- Peter Eliasson, Jan Nordström, Marco Kupiainen (Linköping U., Sweden)
  - Summation-by-parts (LiU SBP)

The Goal: Verification

Verification and Validation

- Verification: Is the code correct?
- Validation: Is the mathematical model appropriate?

Software Development Verification

- Unit testing
- Memory checking
- Static analysis
- Continuous integration

Non-Intrusive Order of Accuracy Component Verification

- Test cases designed to *verify* CFD codes
- Structured quadrilateral and triangle meshes (python scripts)
- Participants required to use provided meshes
  - Mesh independent comparison of codes

Anticipated Order of Accuracy

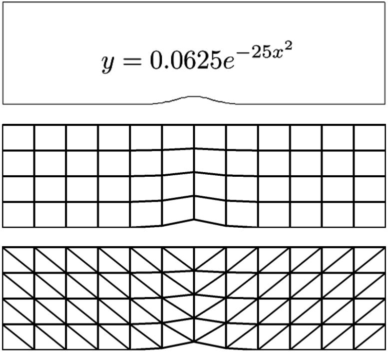
Scheme	L_2 Solution Error	Output Functional Error
Galerkin FEM (CG/DG/HDG)	$O\left(h^{P+1}\right)$	$O\left(h^{2P}\right)$
Finite Volume/Difference	$O\left(h^{P+1}\right)$	$O\left(h^{P+1}\right)$
Non-Galerkin Orthogonal FEM	$O\left(h^{P+1}\right)$	$O\left(h^{P+1}\right)$

Galerkin Finite Element Assumptions

- L_2 Solution Error: $O\left(h^{P+1}\right)$
  - Solution Smoothness: $u \in H^{P+1}$
- Output Functional Error: $O\left(h^{2P}\right)$
  - Solution, Adjoint Smoothness: $u, \psi \in H^{P+1}$
- Accurate Integration
- Adjoint Consistent
- Asymptotic uniform mesh refinement will eventually observe $O\left(h^{2P}\right)$
- Optimal mesh distributions will observe $O\left(h^{2P}\right)$ with fewer DOF

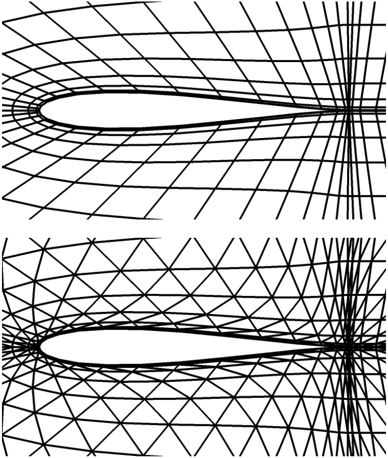
Inviscid Smooth Gaussian Bump

- Used in all previous workshops (but with a variety of meshes)
- Trivial mesh works well
- Verify inviscid fluxes and BC's
- Mach 0.5
- Total pressure and temperature inflow
- Static pressure outflow
- Entropy error: $O\left(h^{P+1}\right)$
    - Mesh is NOT optimized to minimize entropy error



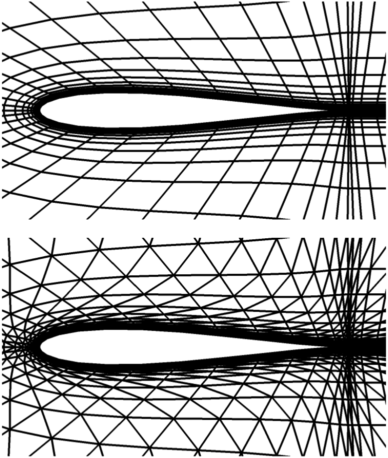
Laminar Joukowski Airfoil

- Verify viscous fluxes and BC's
- Cusped trailing edge
  - Reduce inviscid singularity
- Mach 0.5, $\alpha = 0^\circ$, $Re = 1000$
- Symmetric: $C_l \equiv 0$
- C-grid: Clustering at stagnation point and trailing edge
  - Nested grids with fixed distribution function
  - 6 months of work to find good distribution
- C_d error: $O\left(h^{P+1}\right)$ or $O\left(h^{2P}\right)$



RANS Joukowski Airfoil

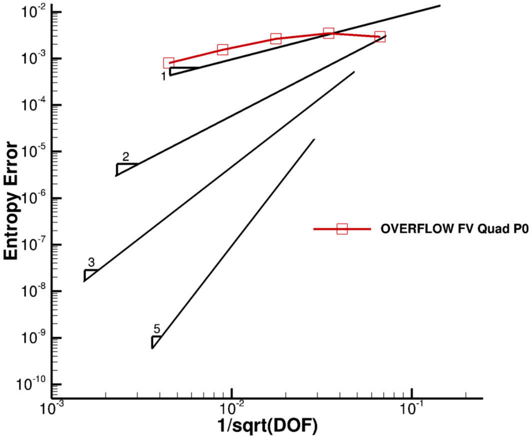
- Verify RANS model and BC's
  - Negative-SA
- Cusped trailing edge
  - Reduce inviscid singularity
- Mach 0.5, $\alpha = 0°$, $Re = 1e6$
- Symmetric: $C_l \equiv 0$
- C-grid: Clustering at stagnation point and trailing edge
  - Nested grids with fixed distribution function
  - 4 months of work to find good distribution
- C_d error: $O\left(h^{P+1}\right)$ or $O\left(h^{2P}\right)$



Is Verification Needed?

OVERFLOW: Smooth Gaussian Bump

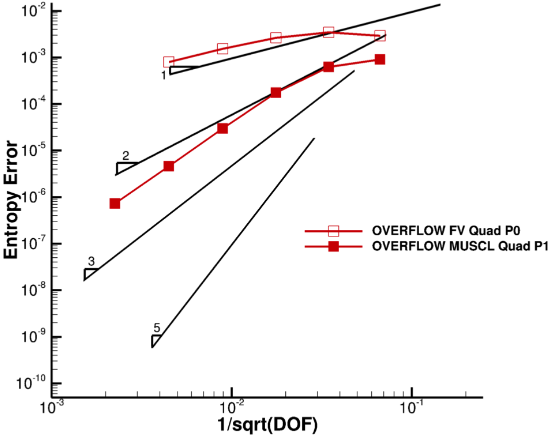
- Highly reputable workhorse CFD code
- Considered to be well *validated*



Is Verification Needed?

OVERFLOW: Smooth Gaussian Bump

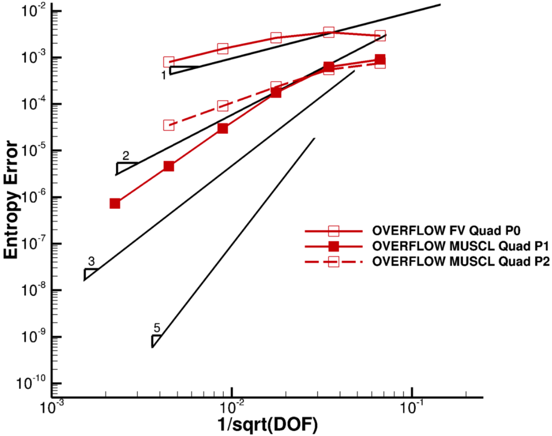
- Highly reputable workhorse CFD code
- Considered to be well *validated*



Is Verification Needed?

OVERFLOW: Smooth Gaussian Bump

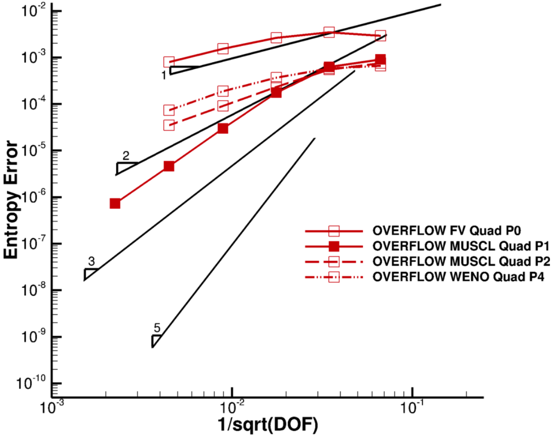
- Highly reputable workhorse CFD code
- Considered to be well *validated*



Is Verification Needed?

OVERFLOW: Smooth Gaussian Bump

- Highly reputable workhorse CFD code
- Considered to be well *validated*



Communication with Participants

Most Common Mistakes

- Using Sutherland's law instead of constant viscosity
- Lack of precision in reference quantities

Other Outcomes

- 2 Groups fixed force calculation due to $C_l \neq 0$
- 1 Group changed force calculation to residual based to get expected order of accuracy
- 1 Group modified BC implementation
- 3 Groups withdrew results concluding bugs in their codes
  - Lack of time to resolve bugs before workshop

Result Caveats

Inviscid Entropy Error

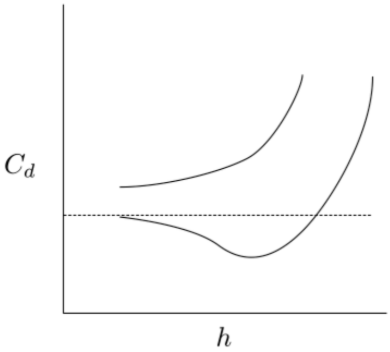
- Norm $\rightarrow$ One sided
- Exact solution is known

Drag Error

- Possible cross over
- No exact solution

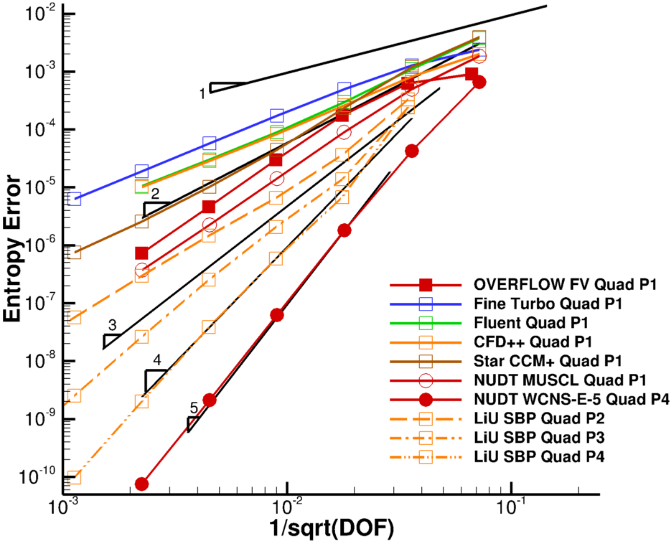
Normalized Time

- (Work Units) $\times$ 5 s
- Represents modern computer



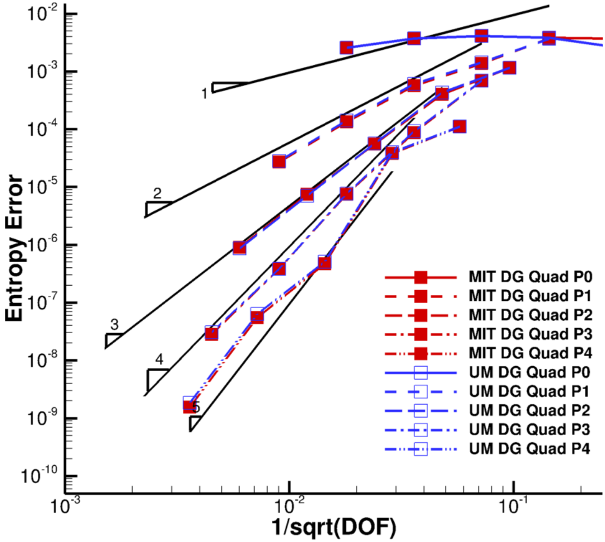
Smooth Gaussian Bump

Finite Volume and Finite Difference



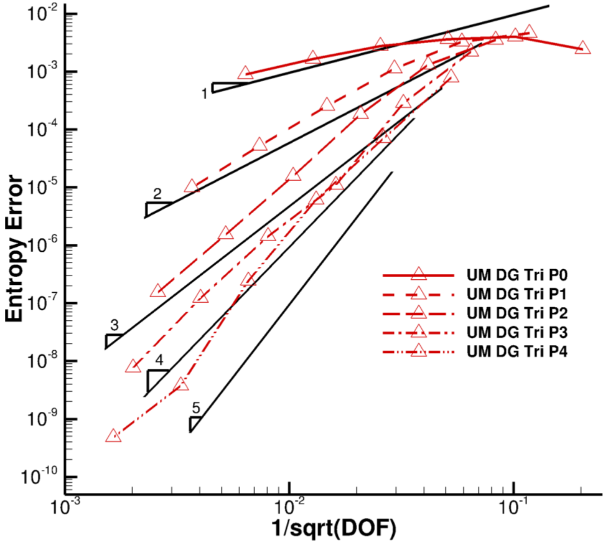
Smooth Gaussian Bump

Discontinuous Galerkin

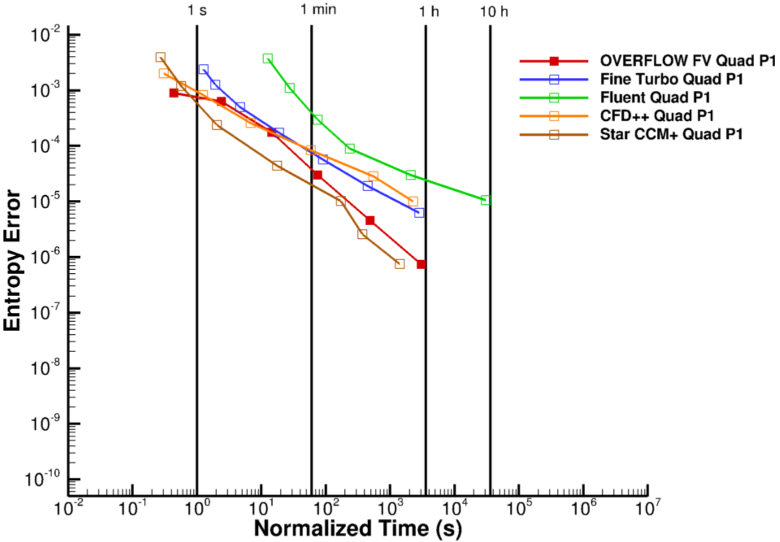


Smooth Gaussian Bump

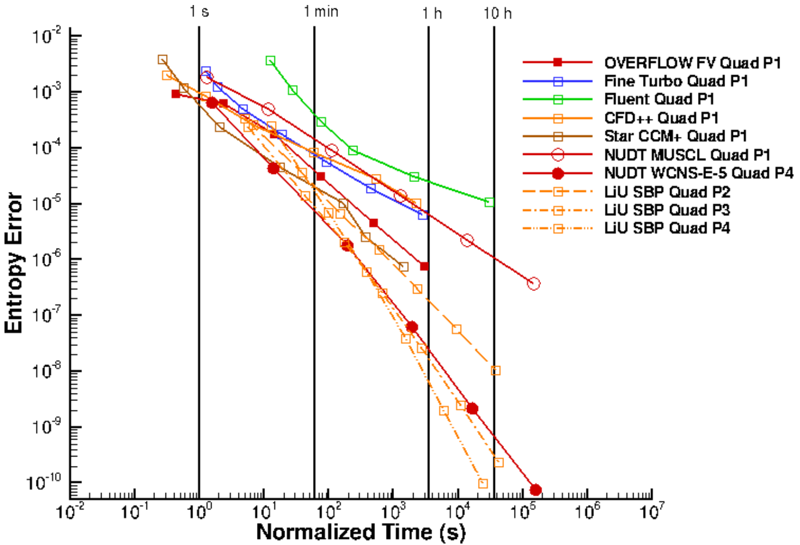
Discontinuous Galerkin



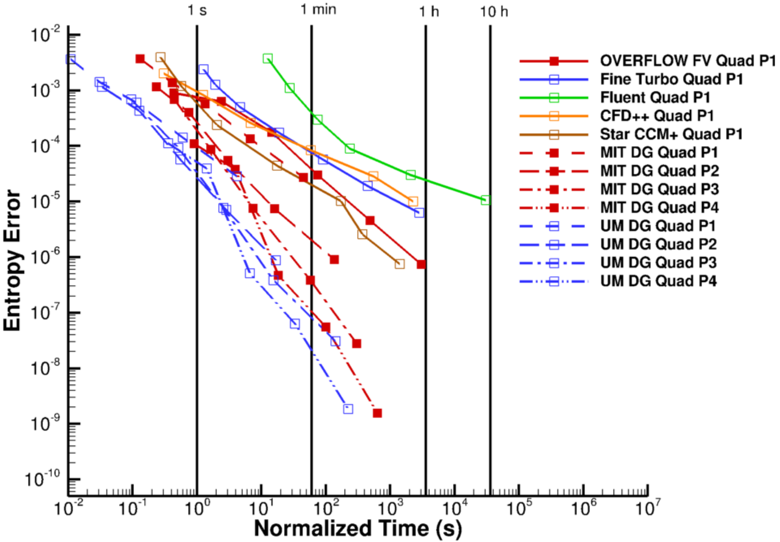
Smooth Gaussian Bump

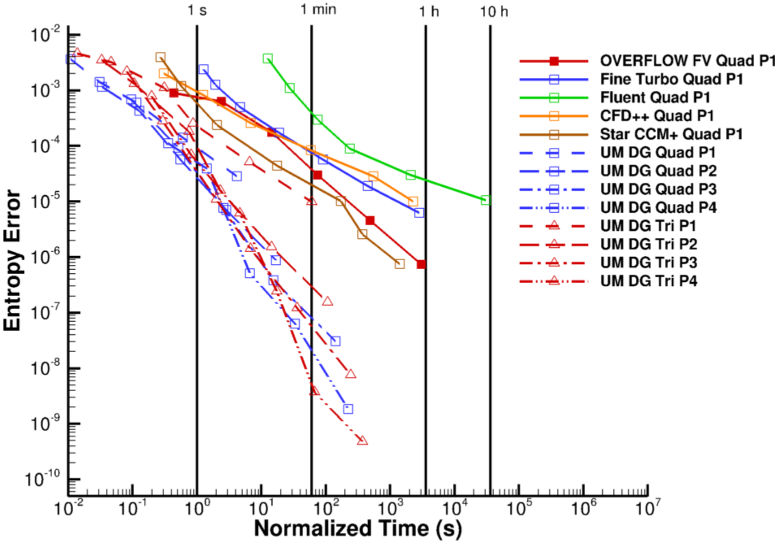


Smooth Gaussian Bump



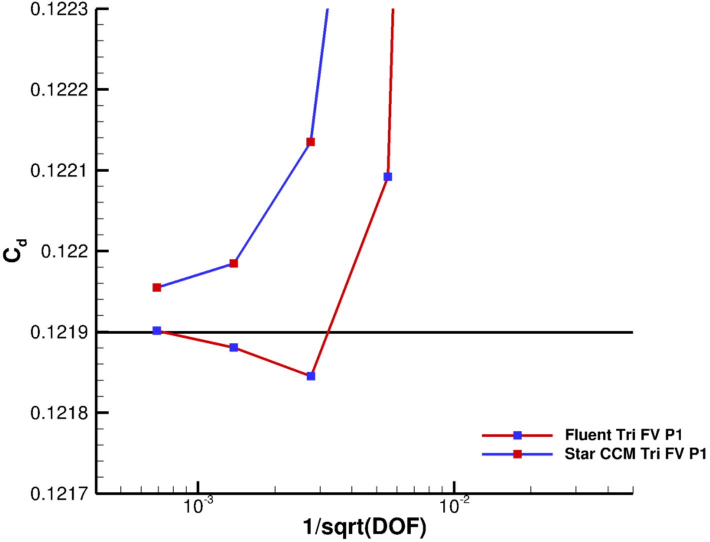
Smooth Gaussian Bump




1 s
1 min
1 h
10 h
Entropy Error
Normalized Time (s)
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7}
OVERFLOW FV Quad P1
Fine Turbo Quad P1
Fluent Quad P1
CFD++ Quad P1
Star CCM+ Quad P1
UM DG Quad P1
UM DG Quad P2
UM DG Quad P3
UM DG Quad P4
UM DG Tri P1
UM DG Tri P2
UM DG Tri P3
UM DG Tri P4

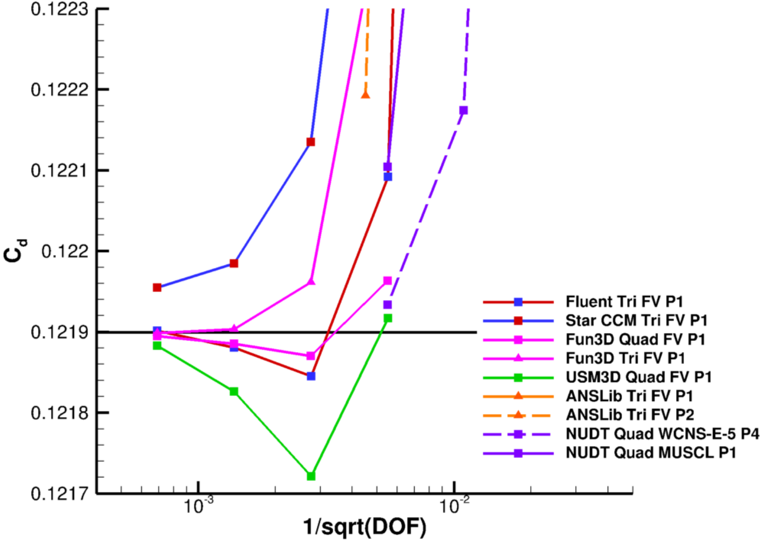
Laminar Joukowski Airfoil

Drag Coefficient



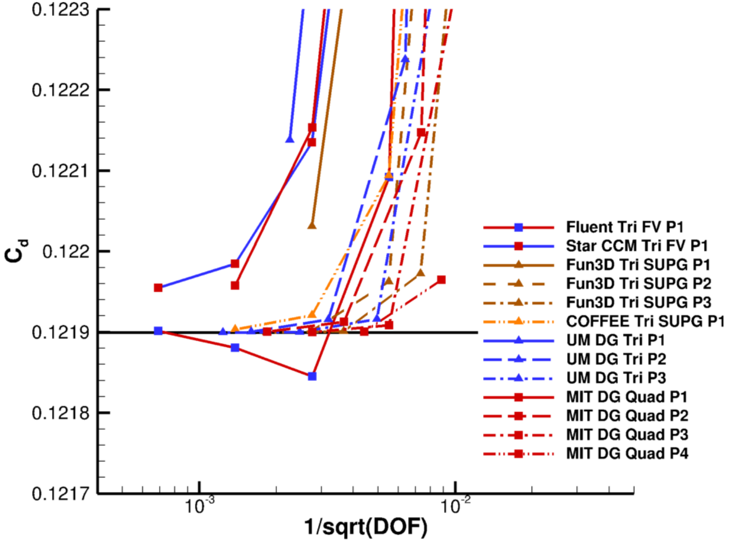
Laminar Joukowski Airfoil

Drag Coefficient



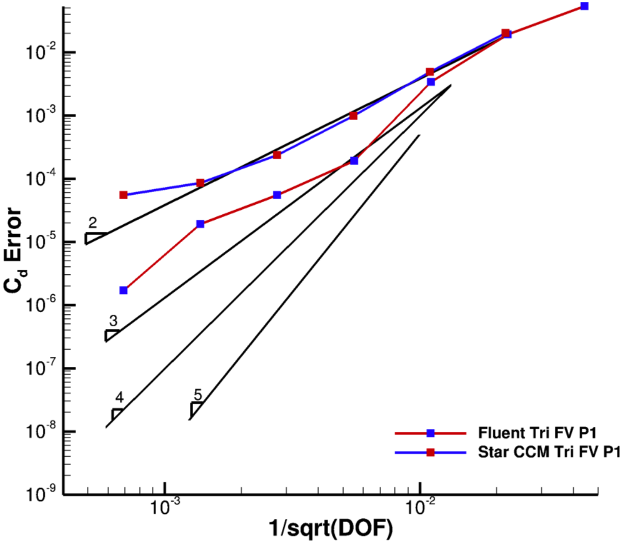
Laminar Joukowski Airfoil

Drag Coefficient



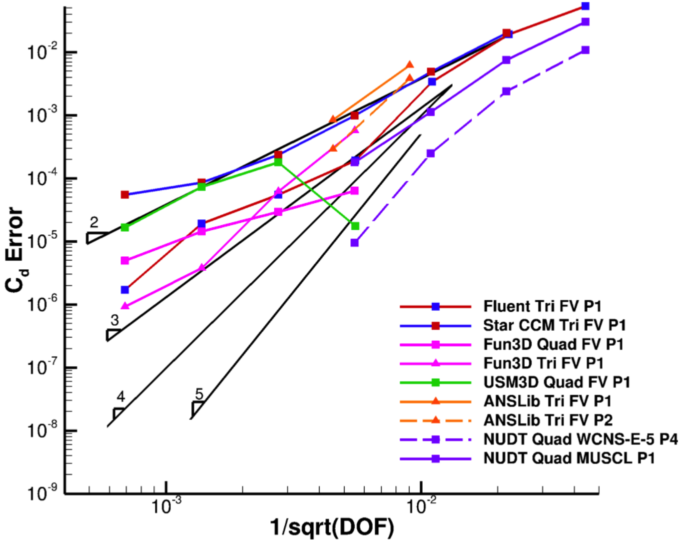
Laminar Joukowski Airfoil

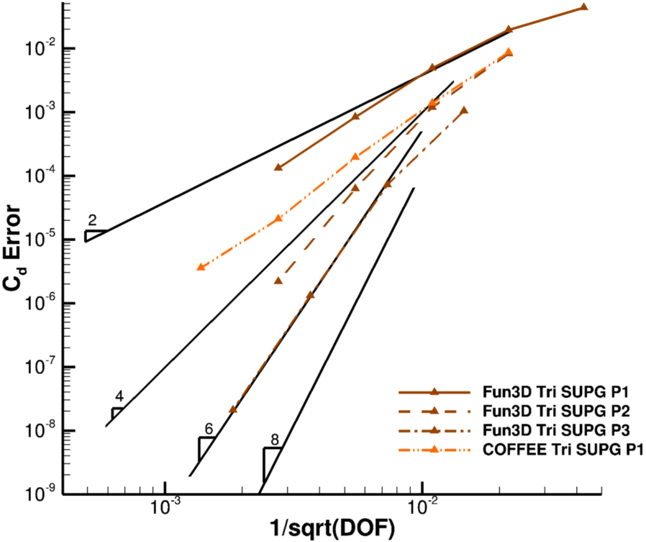
Finite Volume and Finite Difference Order of Accuracy



Laminar Joukowski Airfoil

Finite Volume and Finite Difference Order of Accuracy

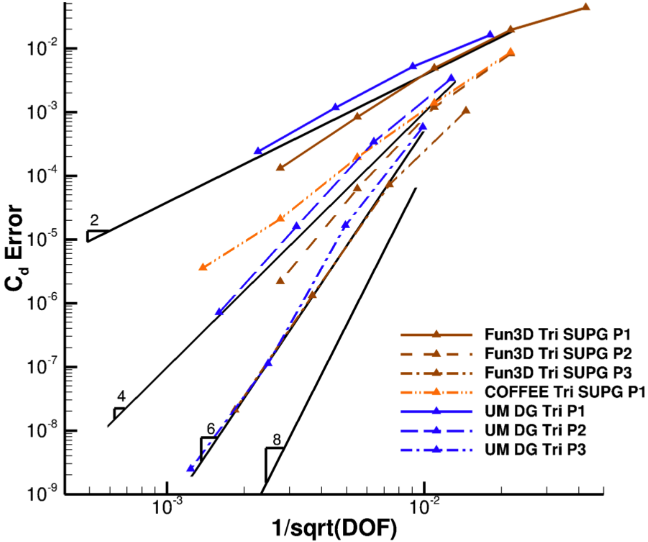


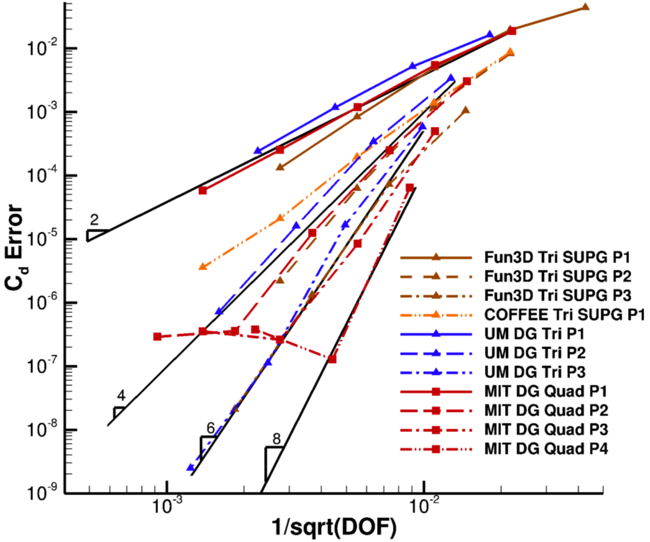


C_d Error
1/sqrt(DOF)
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
2
4
6
8
Fun3D Tri SUPG P1
Fun3D Tri SUPG P2
Fun3D Tri SUPG P3
COFFEE Tri SUPG P1

Laminar Joukowski Airfoil

Galerkin Finite Elements Order of Accuracy

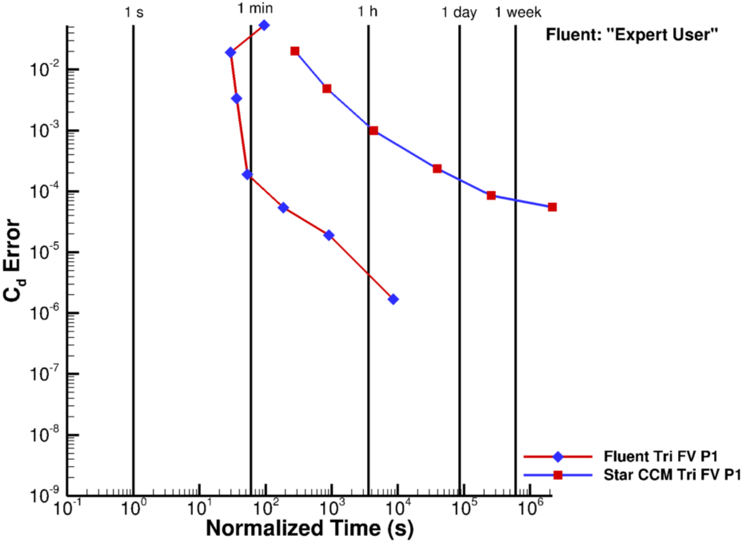


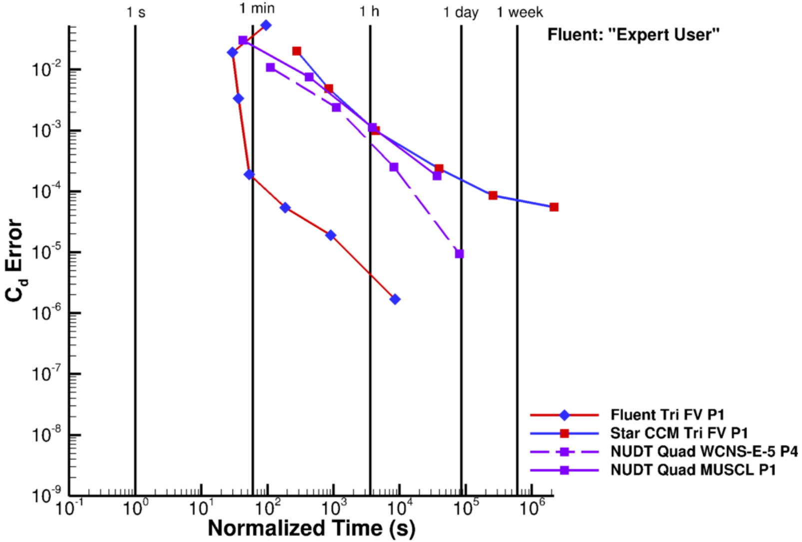


C_d Error
1/sqrt(DOF)
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-3}
10^{-2}
2
4
6
8
Fun3D Tri SUPG P1
Fun3D Tri SUPG P2
Fun3D Tri SUPG P3
COFFEE Tri SUPG P1
UM DG Tri P1
UM DG Tri P2
UM DG Tri P3
MIT DG Quad P1
MIT DG Quad P2
MIT DG Quad P3
MIT DG Quad P4

Laminar Joukowski Airfoil

Timing

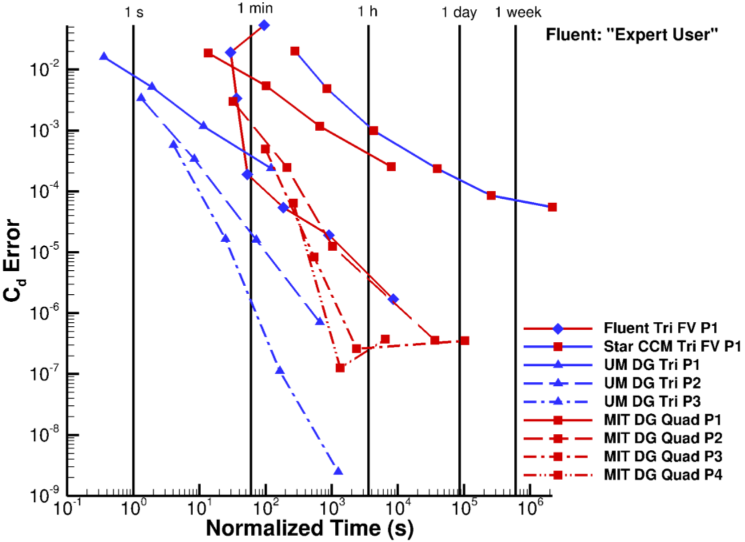




1 s
1 min
1 h
1 day
1 week
Fluent: "Expert User"
C_d Error
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
Normalized Time (s)
Fluent Tri FV P1
Star CCM Tri FV P1
NUDT Quad WCNS-E-5 P4
NUDT Quad MUSCL P1

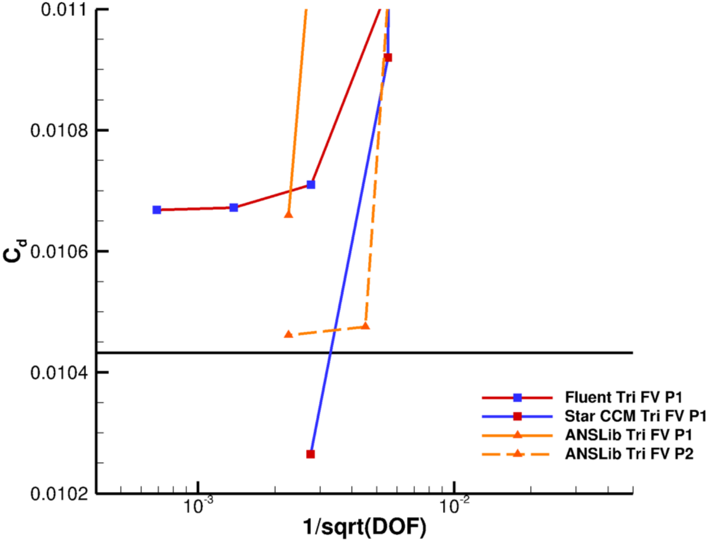
Laminar Joukowski Airfoil

Timing



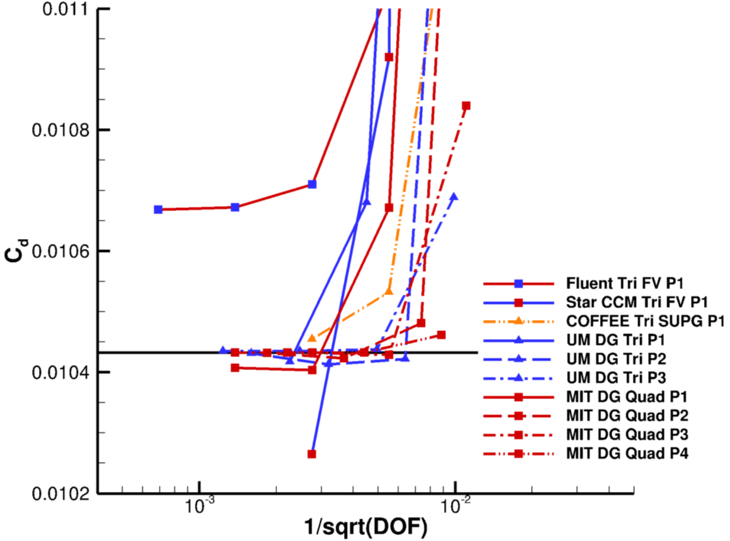
RANS Joukowski Airfoil

Drag Coefficient



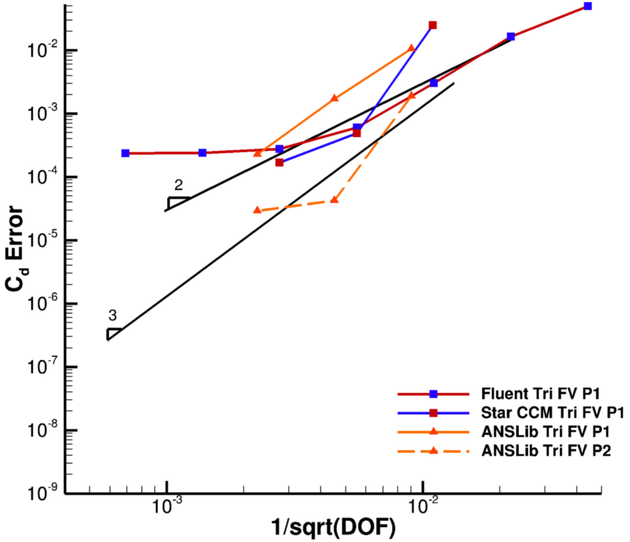
RANS Joukowski Airfoil

Drag Coefficient



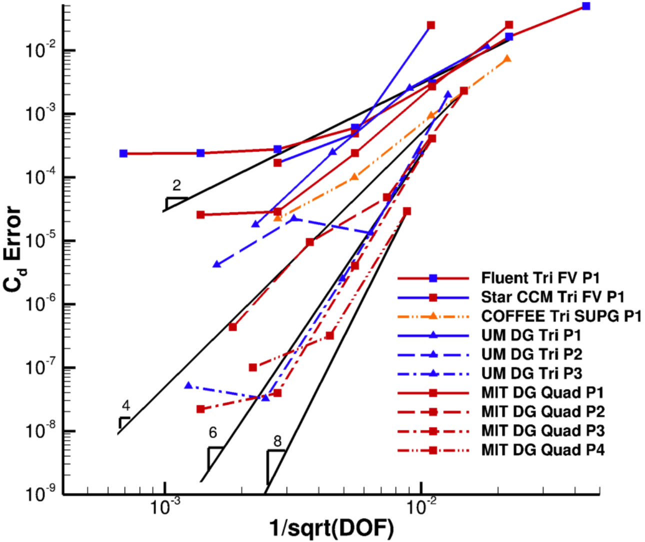
RANS Joukowski Airfoil

Finite Volume and Finite Difference Order of Accuracy



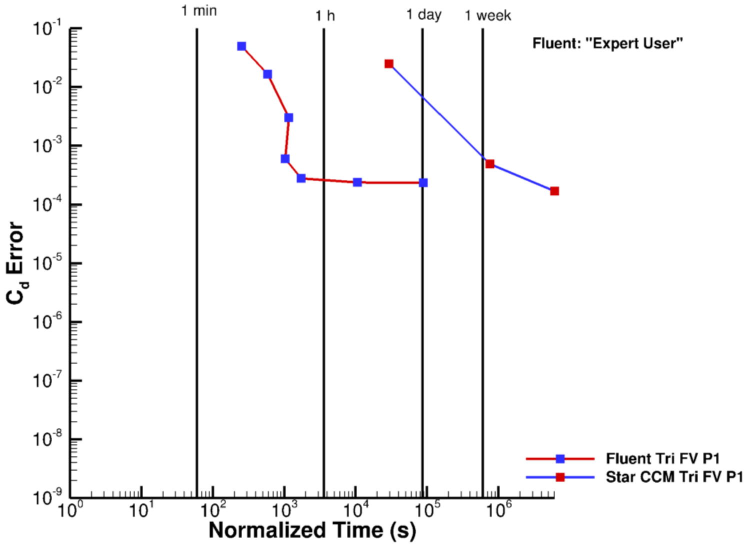
RANS Joukowski Airfoil

Galerkin Finite Elements Order of Accuracy



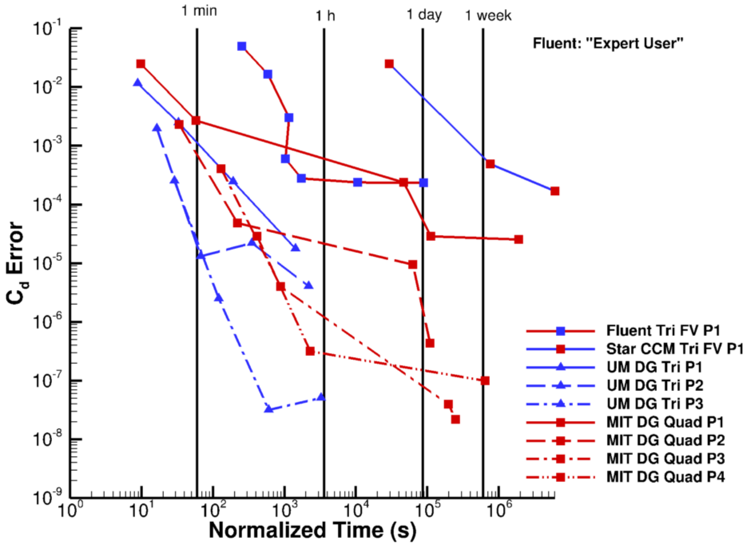
RANS Joukowski Airfoil

Timing



RANS Joukowski Airfoil

Timing



Output Based (Adjoint) Mesh Adaptation

- Contrived problem to alleviate mesh generation
- 6 months to develop grids that observe $O\left(h^{2P}\right)$

Hanging Node or P-Adaptation

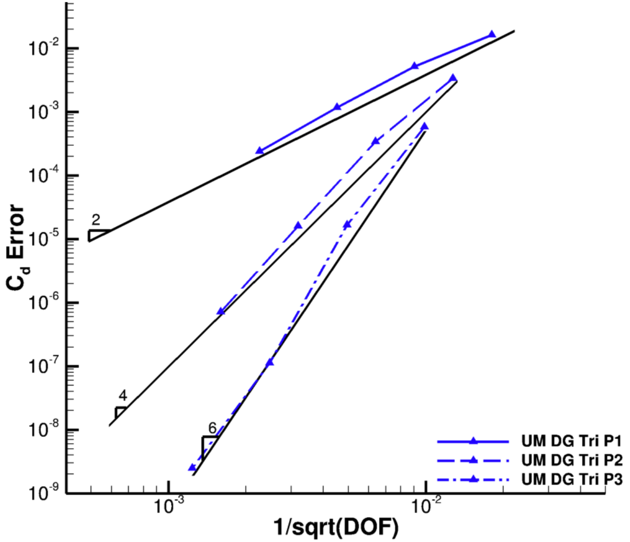
- Local refinement by subdividing or increase polynomial degree of elements
  - Restricted to coarse grid topology
  - Unlikely to observe $O\left(h^{2P}\right)$ error decrease

Mesh Optimization via Error Sampling and Synthesis (MOESS)

- Optimization statement:
  - Find mesh distribution that minimizes output error such that the total DOF count is fixed
- Complete remesh on each adaptation
  - Independent of starting grid topology
  - Often observes $O\left(h^{2P}\right)$ error decrease

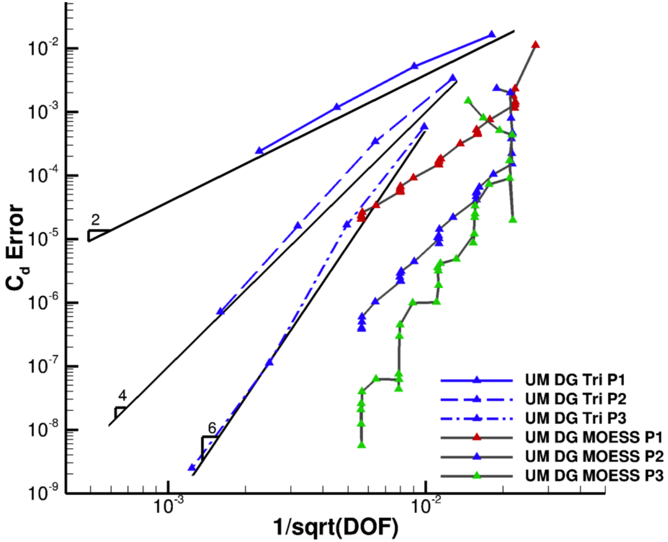
Laminar Joukowski Airfoil

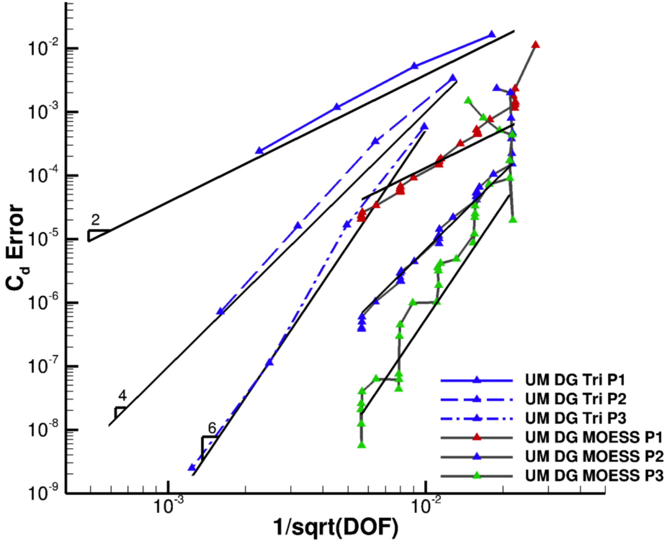
Galerkin Finite Element Mesh Optimization



Laminar Joukowski Airfoil

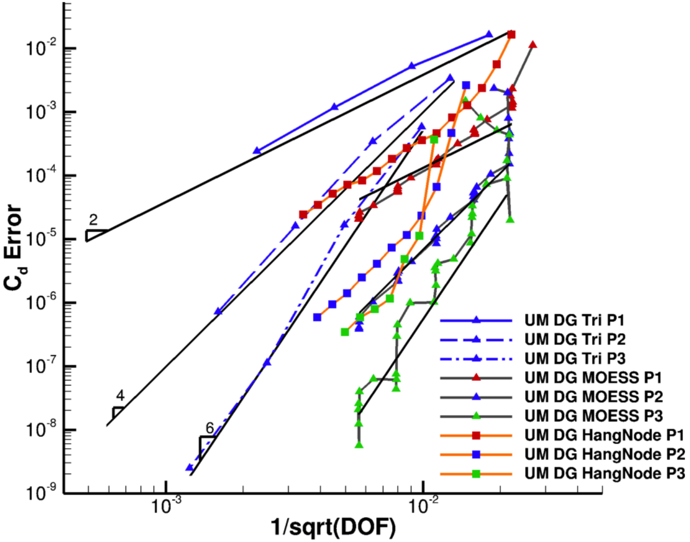
Galerkin Finite Element Mesh Optimization




C_d Error
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-3}
10^{-2}
1/sqrt(DOF)
2
4
6
UM DG Tri P1
UM DG Tri P2
UM DG Tri P3
UM DG MOESS P1
UM DG MOESS P2
UM DG MOESS P3

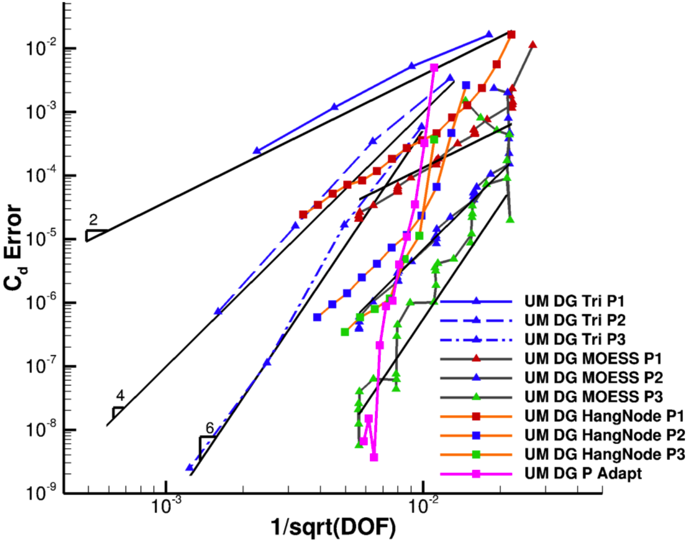
Laminar Joukowski Airfoil

Galerkin Finite Element Mesh Optimization



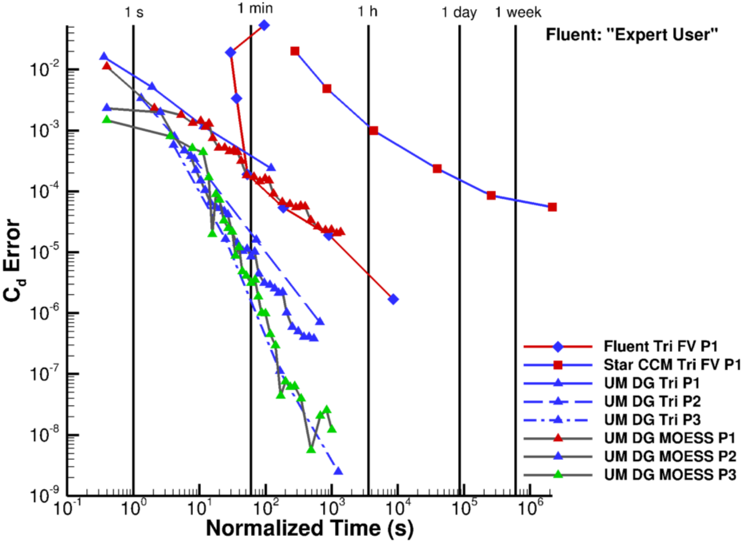
Laminar Joukowski Airfoil

Galerkin Finite Element Mesh Optimization



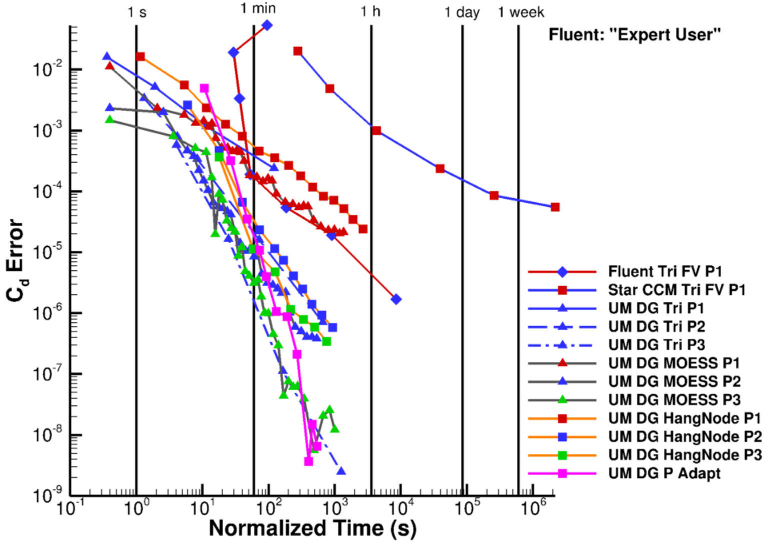
Laminar Joukowski Airfoil

Galerkin Finite Element Mesh Optimization Timing



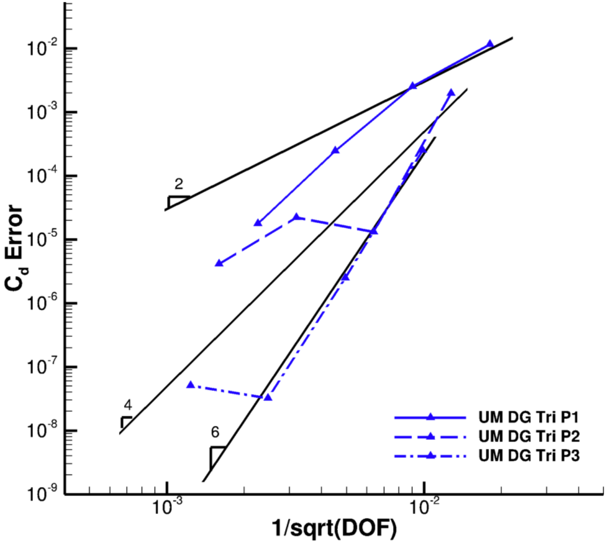
Laminar Joukowski Airfoil

Galerkin Finite Element Mesh Optimization Timing



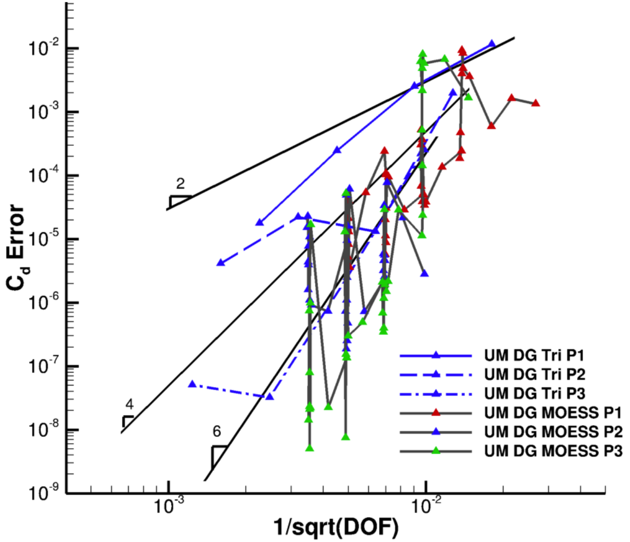
RANS Joukowski Airfoil

Galerkin Finite Element Mesh Optimization



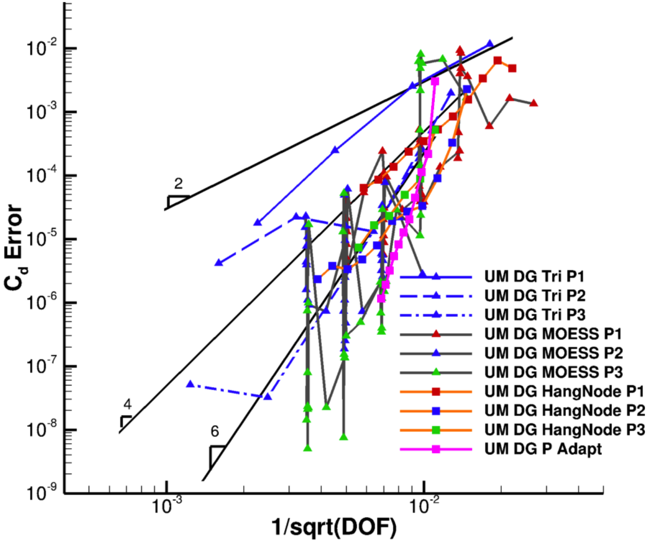
RANS Joukowski Airfoil

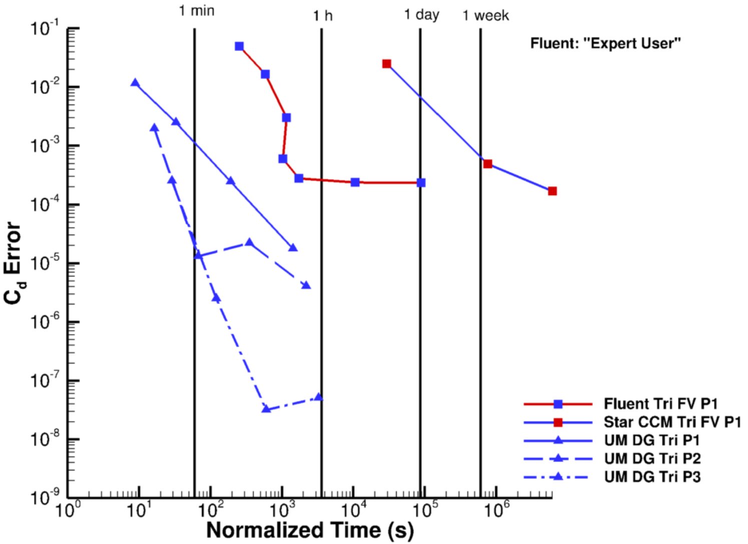
Galerkin Finite Element Mesh Optimization



RANS Joukowski Airfoil

Galerkin Finite Element Mesh Optimization

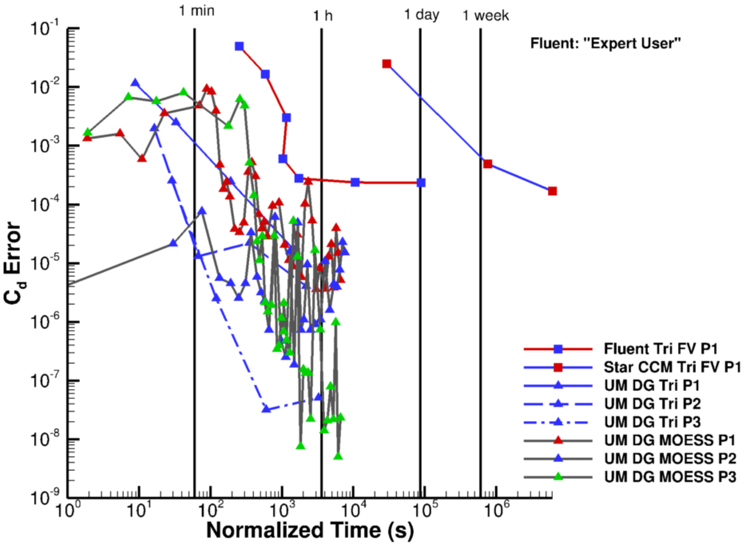




1 min
1 h
1 day
1 week
Fluent: "Expert User"
C_d Error
Normalized Time (s)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
Fluent Tri FV P1
Star CCM Tri FV P1
UM DG Tri P1
UM DG Tri P2
UM DG Tri P3

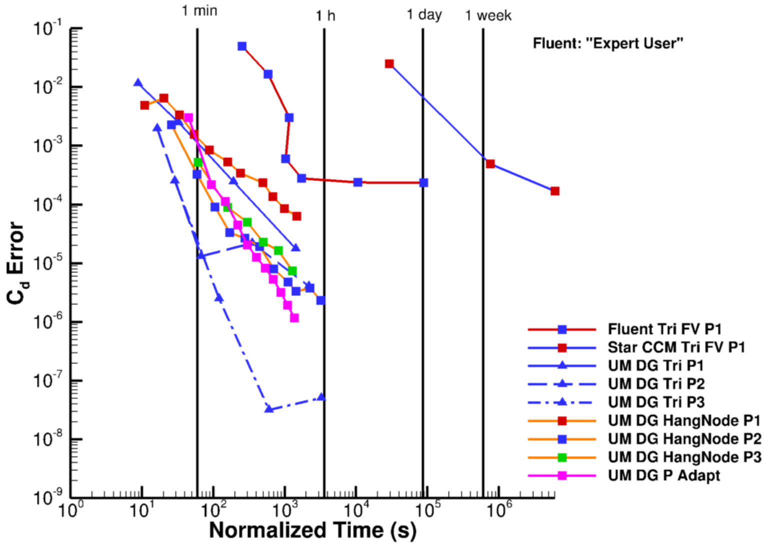
RANS Joukowski Airfoil

Galerkin Finite Element Mesh Optimization Timing



RANS Joukowski Airfoil

Galerkin Finite Element Mesh Optimization Timing



Conclusion

Verification Cases

- Inviscid Smooth Bump
  - No sensitivity to mesh
  - Excellent case
- Laminar
  - Mesh sensitive
  - Gives expected rates
- RANS
  - Mesh sensitive
  - Can give expected rates (needs some work)

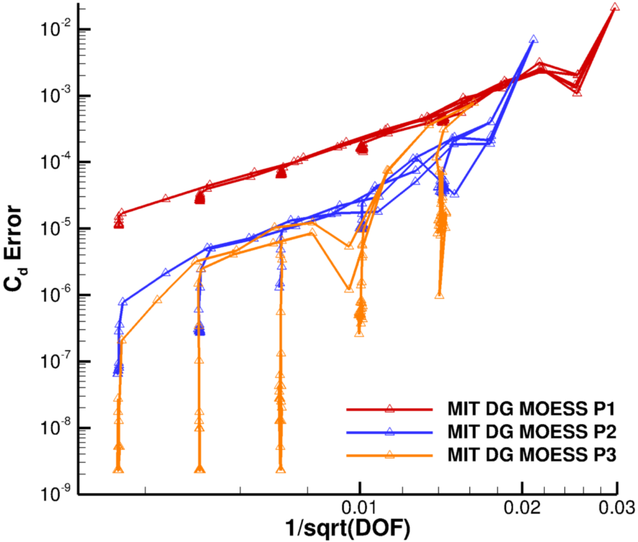
Possible Contributions by HOW4

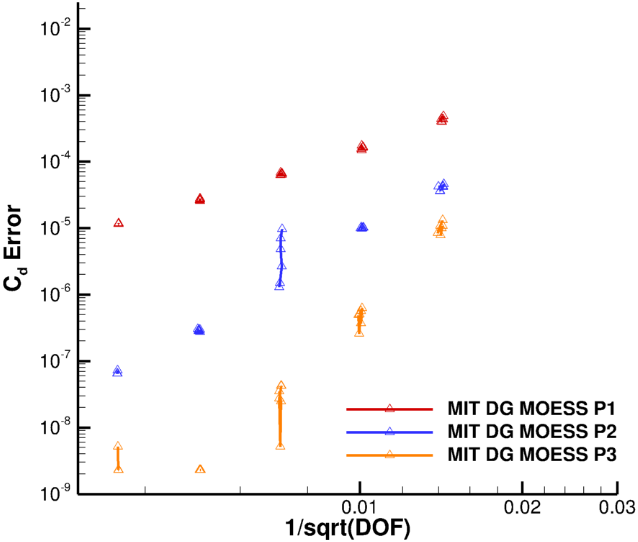
- Provide community with *verification* quality test cases
  - Results and raw data published on website

Backup Slides

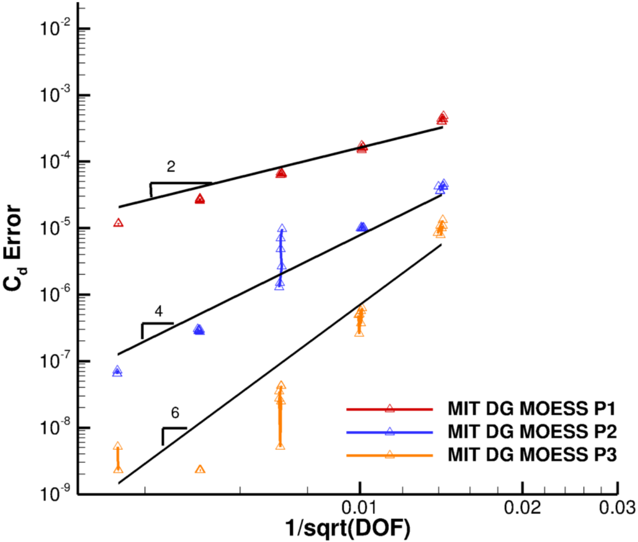
Laminar Joukowski Airfoil

Galerkin Finite Element Mesh Optimization





C_d Error
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
0.01
0.02
0.03
1/sqrt(DOF)
MIT DG MOESS P1
MIT DG MOESS P2
MIT DG MOESS P3



10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
C_d Error
2
4
6
0.01
0.02
0.03
1/sqrt(DOF)
MIT DG MOESS P1
MIT DG MOESS P2
MIT DG MOESS P3