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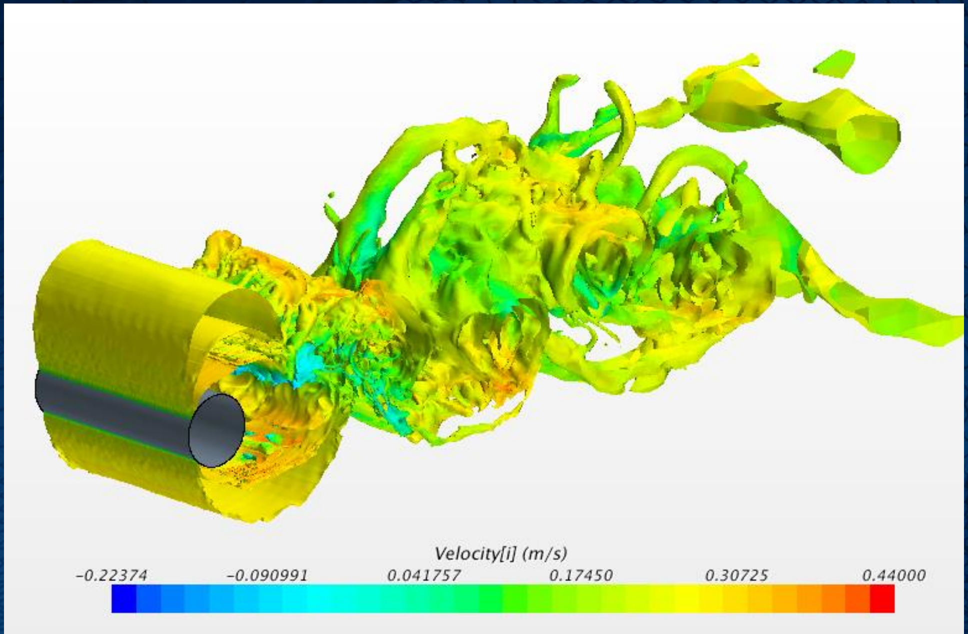



# High-Fidelity Simulation of Unsteady Flow Problems using a 3rd Order Hybrid MUSCL/CD scheme

ECCOMAS, June 6th-11th 2016, Crete Island, Greece

**A. West & D. Caraeni**

# Outline

- Industrial Motivation

- Numerical Schemes

- Test Cases:
    - Inviscid Vortex Transport
    - DNS of Taylor-Green Vortices at Re = 1,600
    - LES of a Circular Cylinder at Re =3,900

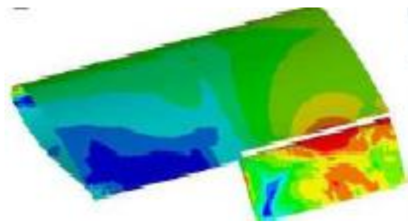
- Conclusions

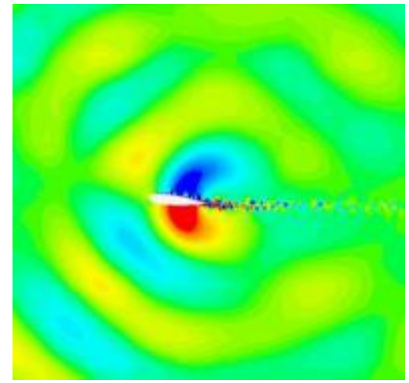
# Industrial Motivation

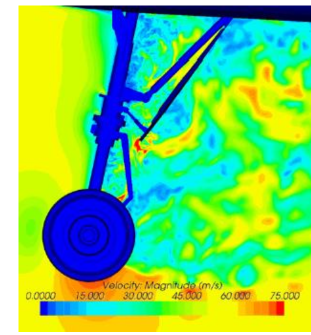
## High-Order CFD targeted for high fidelity simulations

***Aerospace***

- *wing transition,*
- *high-lift devices*
- *engine noise*
- *landing gear aeroacoustics*

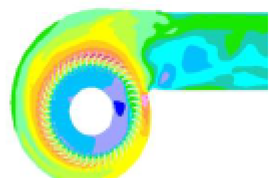


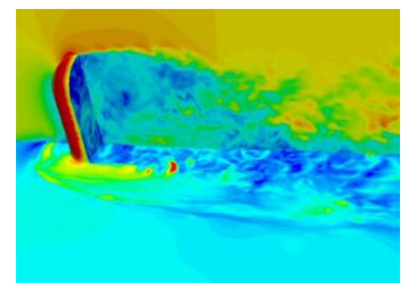





***Automobiles/Trucks***

- *full vehicle aerodynamics,*
- *mirror, window, sunroof aeroacoustics,*
- *HVAC fans, ducts, nozzles, turbochargers*





***Combustion***

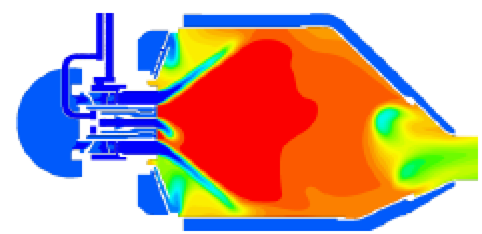
- *gas turbine, reciprocating engine*

***Nuclear*** *(steam line/T-junctions, etc.)*

***Wind turbines***

***Numerical solution of (unsteady) turbulent flows requires:***

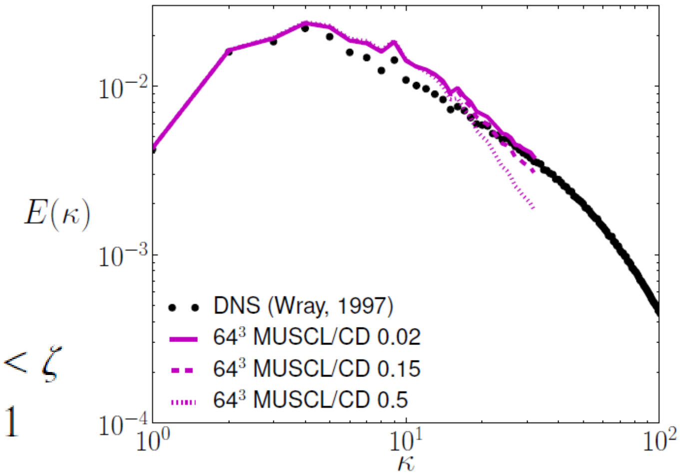
- ***Accuracy ( + energy conservation, realizability)***
- ***Robustness (large industrial meshes)***
- ***Efficiency (CPU, 10-100 k cores)***



# Numerical schemes

- STAR-CCM+, cell-centered FV solver of arbitrary cells
- Two Solvers:
  - Coupled approximate-Riemann solver for all flow regimes
  - Segregated Rhie-Chow SIMPLE or PISO based solver
- Least-Squares gradient reconstruction with high-fidelity gradient limiters
- Preconditioning and other algorithm enhancements for high-fidelity simulations
- BDF 2 and optimized BDF2 (with reduced numerical error) implemented
- LES models (WALE, Smagorinsky and dynamic Smagorinsky)

- Convective flux:
  - $2^{nd}$ Order Upwind scheme
  - $2^{nd}$ Order Bounded-CD (hybrid)
  - $3^{rd}$ Order MUSCL/CD scheme (hybrid)



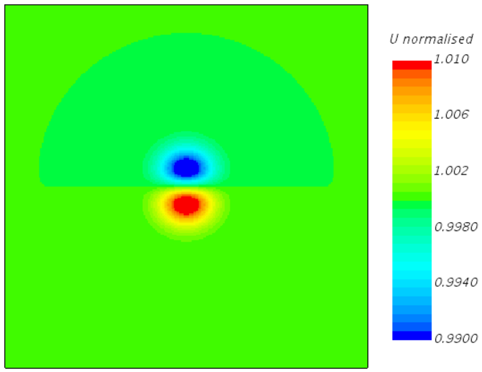

$$(\dot{m}\phi)_f = \begin{cases} \dot{m}\phi_{FOU} & \text{for } \zeta < 0 \text{ or } 1 < \zeta \\ \dot{m}\left(\sigma\phi_{MUSCL3} + (1-\sigma)\phi_{CD3}\right) & \text{for } 0 \le \zeta \le 1 \end{cases}$$

blending based on local smoothness indicator via Normalised Variable Diagram (Darwish et al.)

# Inviscid Vortex Transport

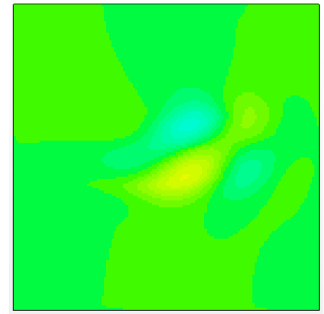
"Slow vortex": $M_\infty = 0.05$, $\beta = 1/50$, $R = 0.005$.

**Initial Solution T=0**

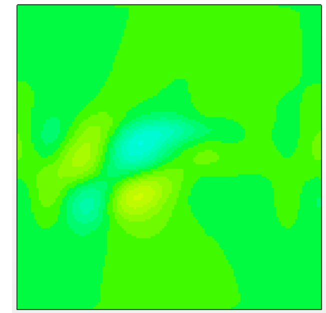



$$
\begin{aligned}
\delta u &= -(U_\infty \beta)\frac{y - Y_c}{R}\exp\left(\frac{-r^2}{2}\right)\\
\delta v &= (U_\infty \beta)\frac{x - X_c}{R}\exp\left(\frac{-r^2}{2}\right)\\
\delta T &= \frac{1}{2C_p}(U_\infty \beta)^2 \exp(-r^2)\\
u_0 &= U_\infty + \delta u\\
v_0 &= \delta v\\
C_p &= \frac{\gamma}{\gamma - 1} R_{\mathrm{gas}}\\
r &= \frac{\sqrt{(x - X_c)^2 + (y - Y_c)^2}}{R}\\
U_\infty &= M_\infty \sqrt{\gamma R_{\mathrm{gas}} T_\infty}.
\end{aligned}
$$

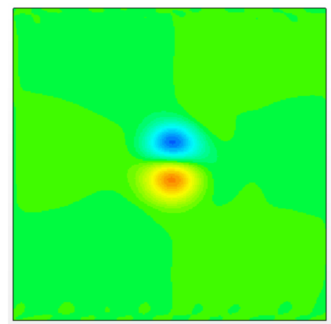
**Solution T= 50**



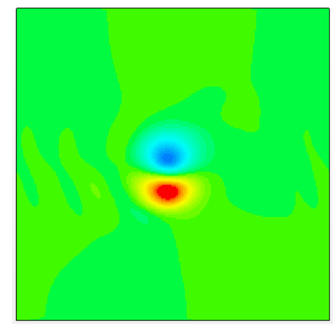
$128^2$ $2^{nd}$-O upwind



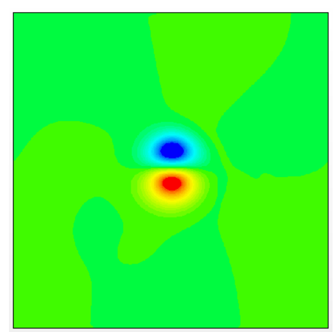
$128^2$ $3^{rd}$-O MUSCL/CD



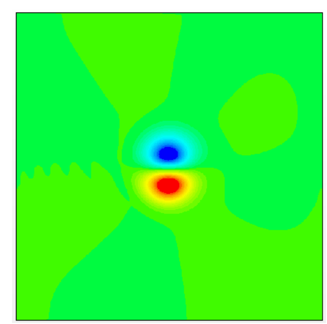
$256^2$ $2^{nd}$-O upwind



$256^2$ $3^{rd}$-O MUSCL/CD



$512^2$ $2^{nd}$-O upwind



$512^2$ $3^{rd}$-O MUSCL/CD

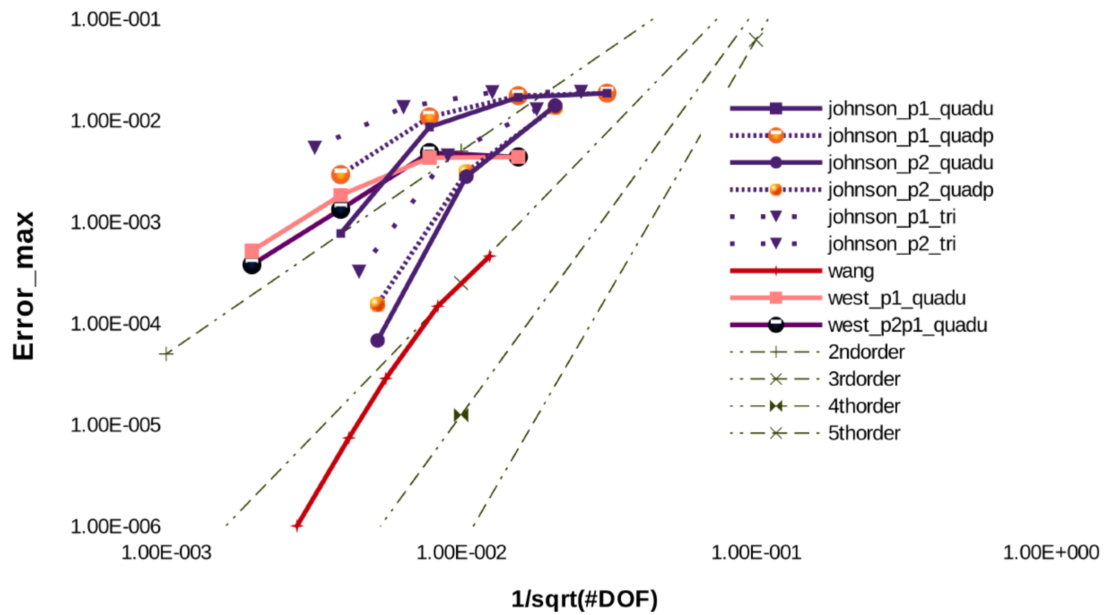
# Inviscid Vortex Transport

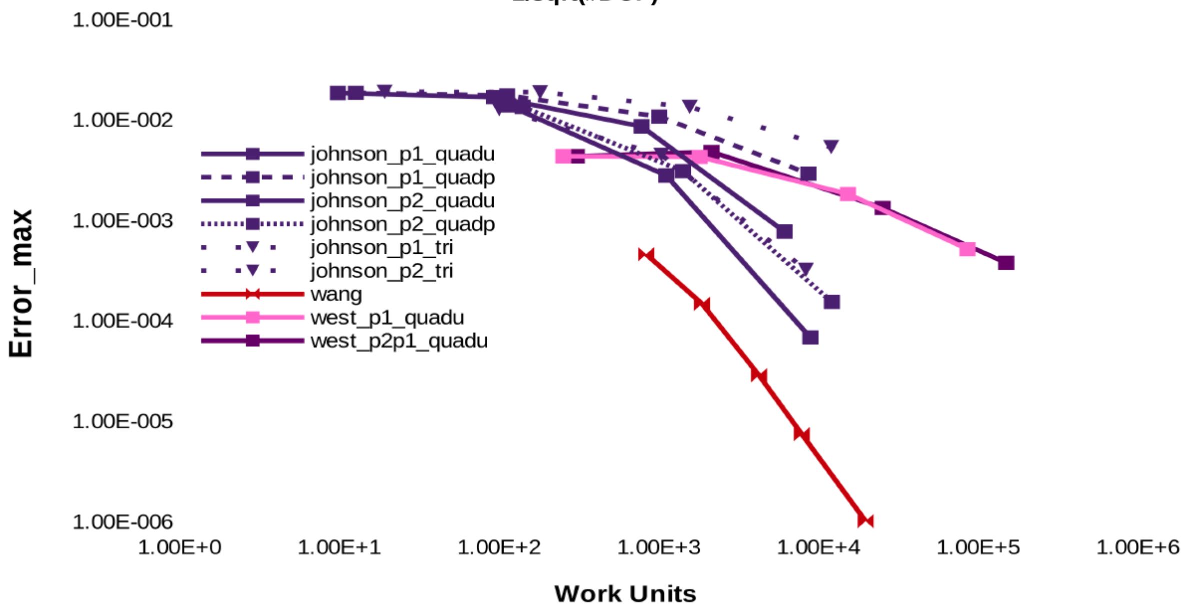
$$L_2Error_{|U|} = \sqrt{\frac{\sum_{i=1}^{N}\int_{V_i}(|U| - |U|_{initial})^2 dV}{\sum_{i=1}^{N}\int_{V_i} dV}}$$

$$|U| = \sqrt{u^2 + v^2}$$

$$WorkUnit = \frac{CPUs \times t_{STAR-CCM+}}{t_{TauBench}}$$

$$h = \frac{1}{\sqrt{nDOF}} = \frac{1}{\sqrt{N}}$$



# DNS of Taylor-Green Vortices at Re=1,600

Evolution of quantities:

$$E_k = \frac{1}{\rho_0\,\Omega}\int_\Omega \rho\,\frac{\mathbf{v}\cdot\mathbf{v}}{2}\,d\Omega\,.$$

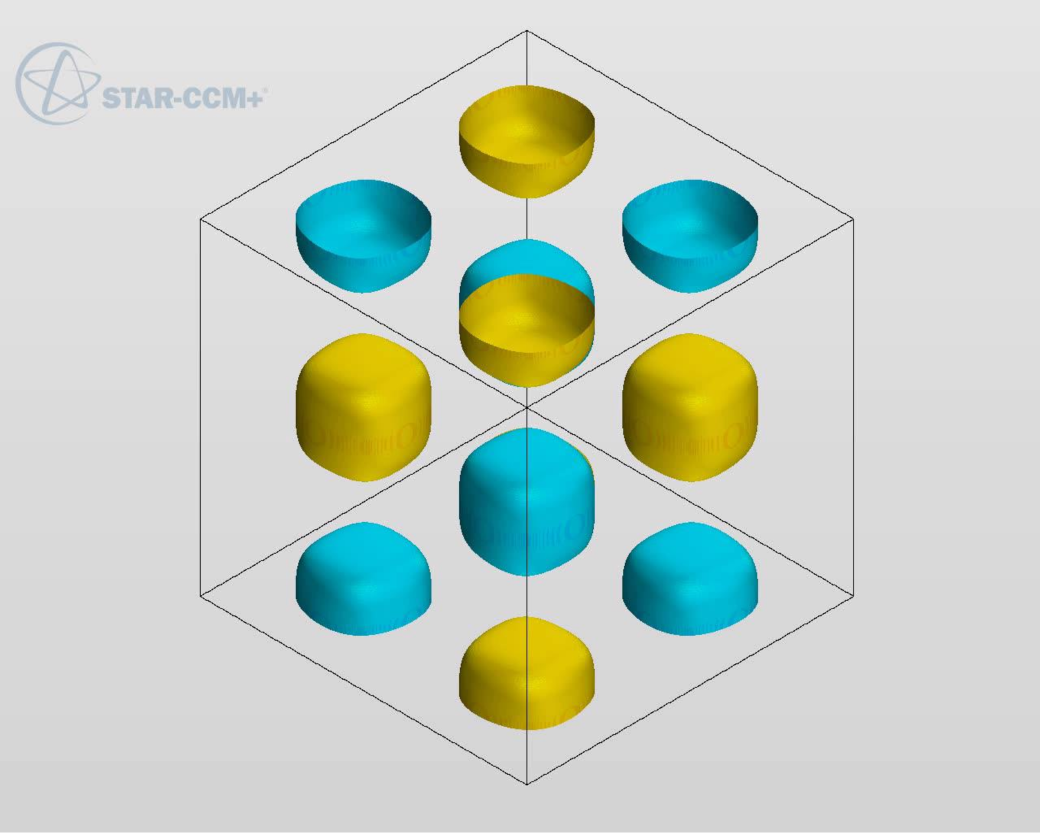
$$\epsilon = -\frac{dE_k}{dt}.$$

$$\mathcal{E} = \frac{1}{\rho_0\,\Omega}\int_\Omega \rho\,\frac{\omega\cdot\omega}{2}\,d\Omega\,.$$

Initial Conditions:

$$\begin{aligned}
u &= V_0 \sin\left(\frac{x}{L}\right)\cos\left(\frac{y}{L}\right)\cos\left(\frac{z}{L}\right)\,,\\
v &= -V_0 \cos\left(\frac{x}{L}\right)\sin\left(\frac{y}{L}\right)\cos\left(\frac{z}{L}\right)\,,\\
w &= 0\,,\\
p &= p_0 + \frac{\rho_0 V_0^2}{16}\left(\cos\left(\frac{2x}{L}\right)+\cos\left(\frac{2y}{L}\right)\right)\left(\cos\left(\frac{2z}{L}\right)+2\right)\,.
\end{aligned}$$

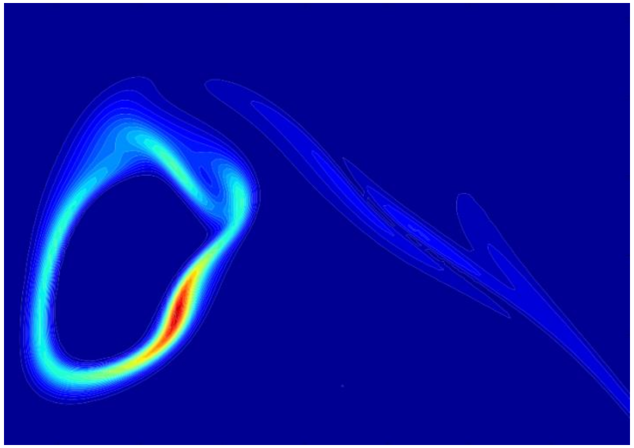
Isosurface of vorticity magnitude coloured by vertical vorticity



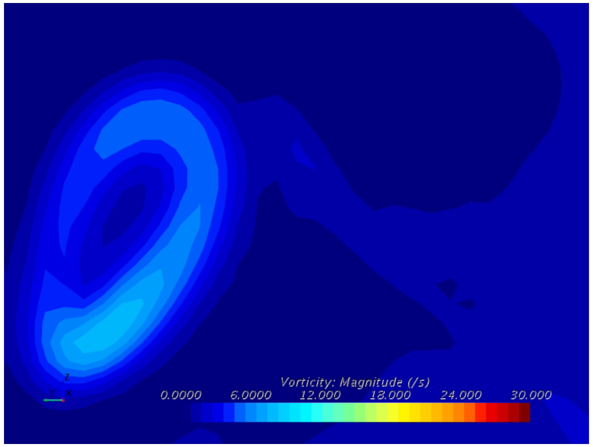

Animation from Patrick McGah

# DNS of Taylor-Green Vortices at Re=1,600

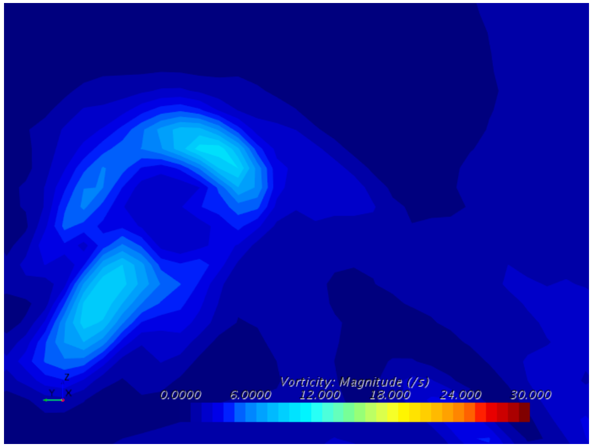
Contours of dimensionless vorticity magnitude $|\omega| L/V_0$ at $x= -\pi L$ and $t_c = 8$.



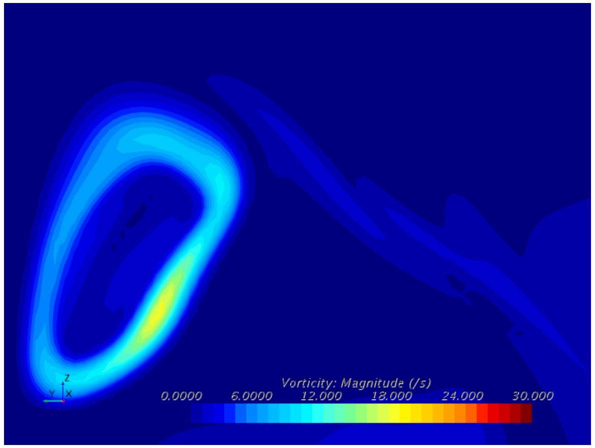
Spectral DNS
(Van Rees et. al, 2011)



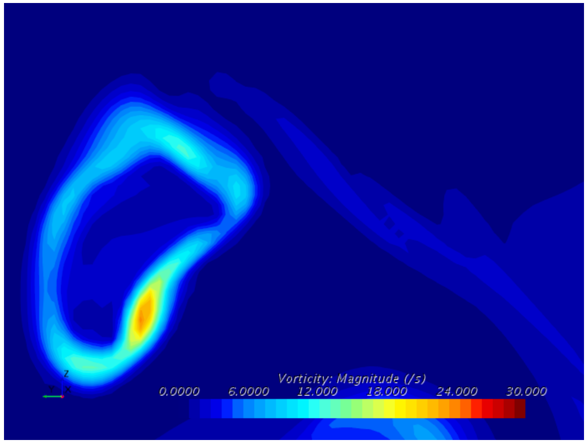

$128^3$ $2^{nd}$-O upwind




$128^3$ $3^{rd}$-O MUSCL/CD



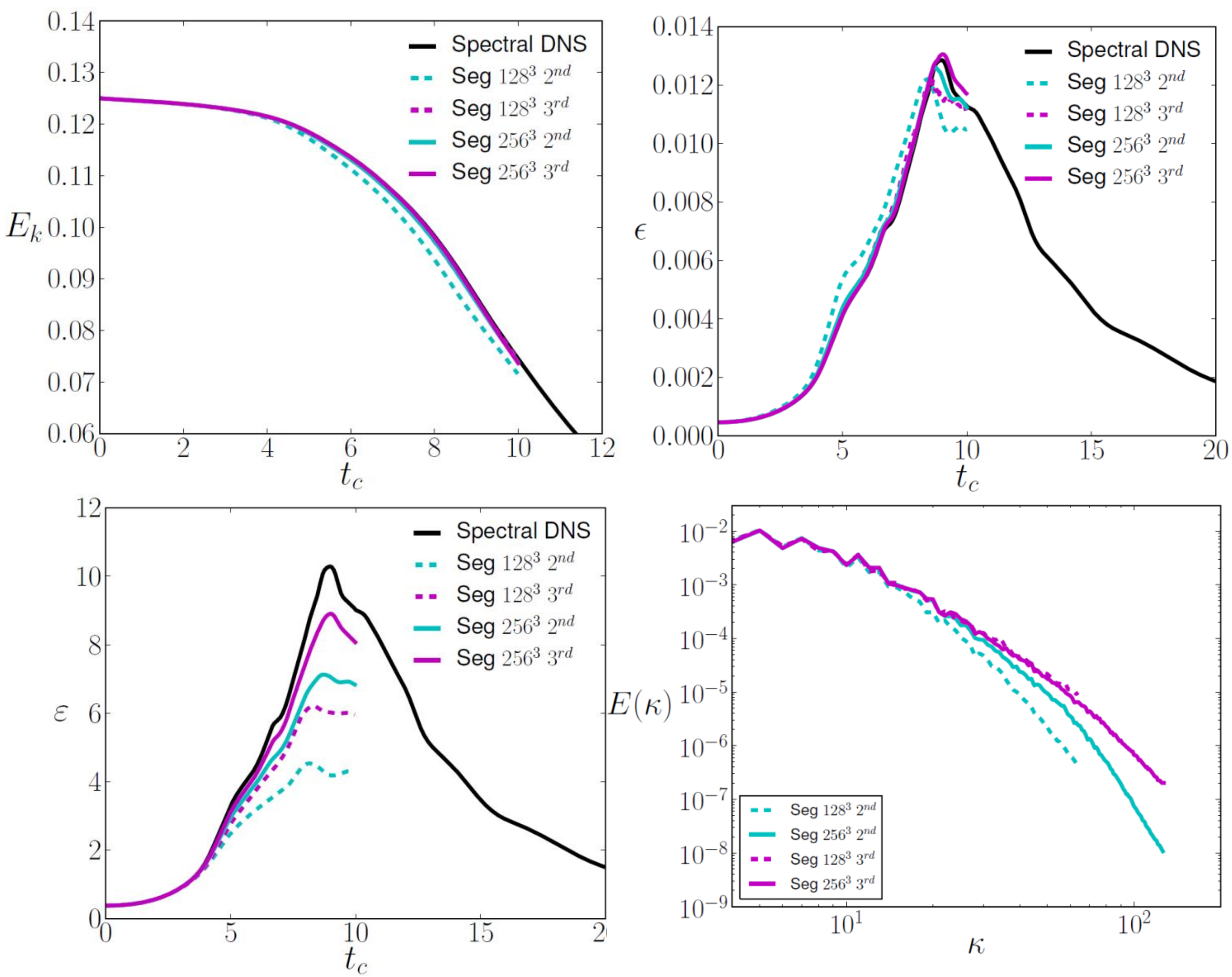

$256^3$ $2^{nd}$-O upwind




$256^3$ $3^{rd}$-O MUSCL/CD

# DNS of Taylor-Green Vortices at Re=1,600

Temporal evolution of total kinetic energy, energy dissipation rate and enstrophy dissipation



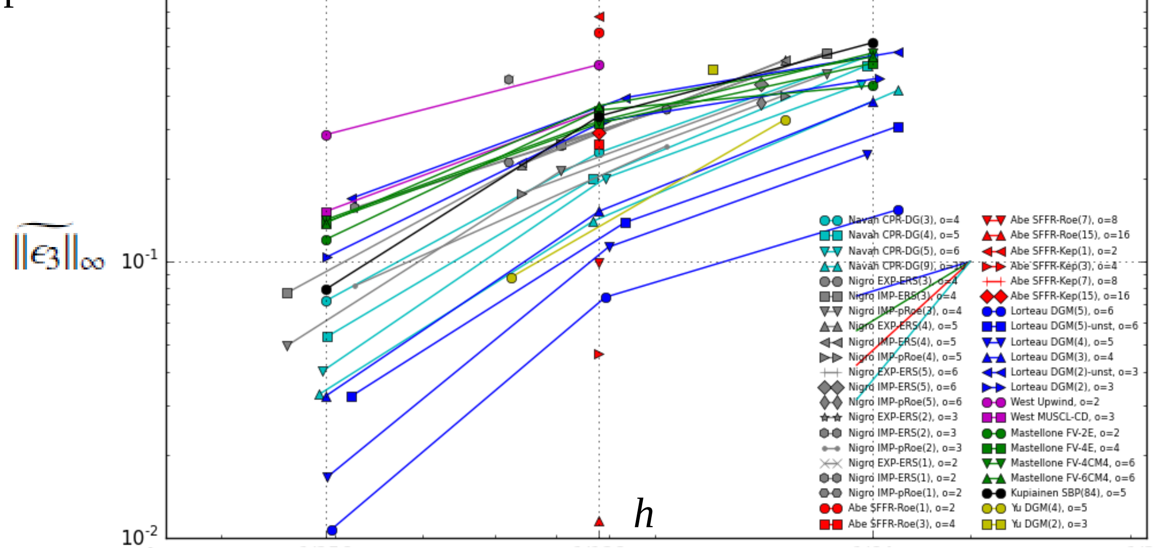

$$E_k = \frac{1}{\rho_0\,\Omega}\int_\Omega \rho\,\frac{\mathbf{v}\cdot\mathbf{v}}{2}\,d\Omega\,.$$

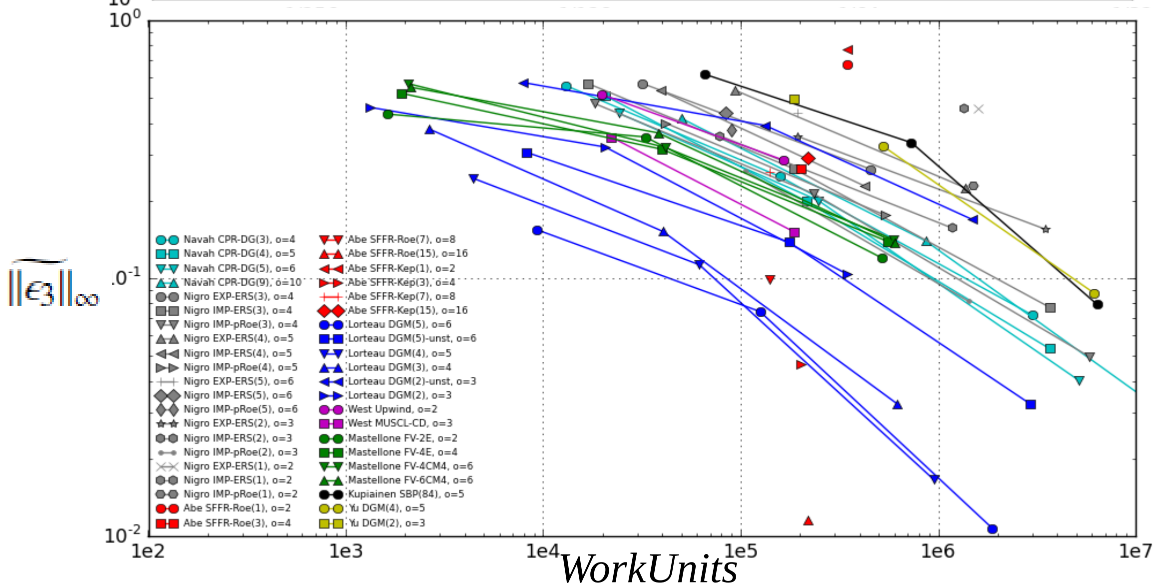
$$\epsilon = -\frac{dE_k}{dt}.$$

$$\mathcal{E} = \frac{1}{\rho_0\,\Omega}\int_\Omega \rho\,\frac{\omega\cdot\omega}{2}\,d\Omega\,.$$

# DNS of Taylor-Green Vortices at Re=1,600

$L_\infty$ error on energy dissipation rate, enstrophy and difference between measured and theoretical dissipation




$$\widetilde{\|\epsilon_1\|}_\infty = \frac{\left\| \frac{dE_k}{dt} - \left(\frac{dE_k}{dt}\right)^\star \right\|_{t,\infty}}{max\left(\frac{dE_k}{dt}\right)^\star},$$

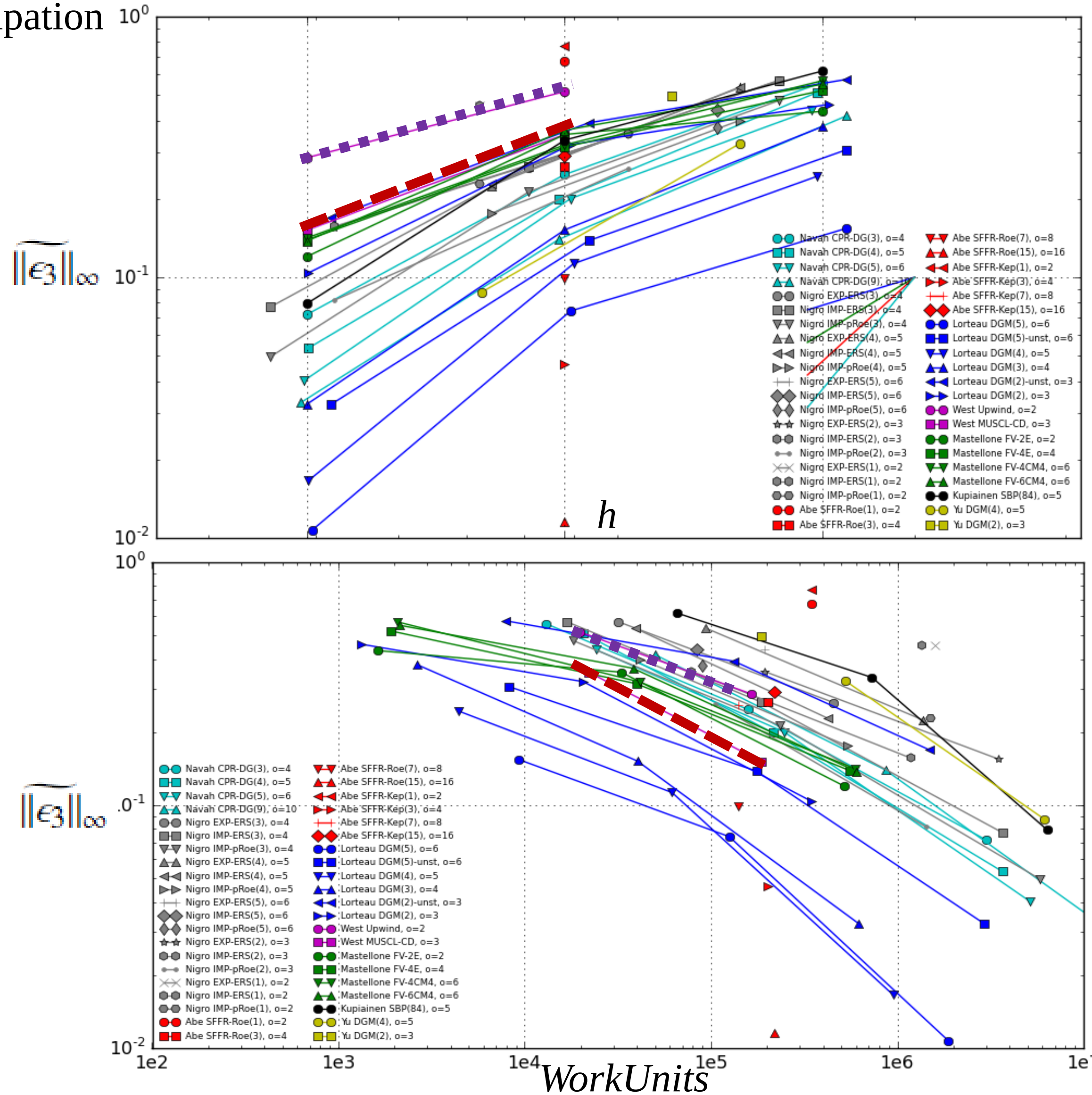
$$\widetilde{\|\epsilon_2\|}_\infty = \frac{\left\| 2\nu\varepsilon - 2\nu\varepsilon^\star \right\|_{t,\infty}}{max\left(\frac{dE_k}{dt}\right)^\star},$$

$$\widetilde{\|\epsilon_3\|}_\infty = \frac{\left\| \frac{dE_k}{dt} - 2\nu\varepsilon \right\|_{t,\infty}}{max\left(\frac{dE_k}{dt}\right)^\star},$$

$$\|a\|_{t,\infty} = \max_{\in[0,10]} |a|.$$

# DNS of Taylor-Green Vortices at Re=1,600

$L_\infty$ error on energy dissipation rate, enstrophy and difference between measured and theoretical dissipation




$$\widetilde{\|\epsilon_1\|}_\infty = \frac{\left\| \frac{dE_k}{dt} - \left(\frac{dE_k}{dt}\right)^\star \right\|_{t,\infty}}{max\left(\frac{dE_k}{dt}\right)^\star},$$

$$\widetilde{\|\epsilon_2\|}_\infty = \frac{\left\| 2\nu\varepsilon - 2\nu\varepsilon^\star \right\|_{t,\infty}}{max\left(\frac{dE_k}{dt}\right)^\star},$$

$$\widetilde{\|\epsilon_3\|}_\infty = \frac{\left\| \frac{dE_k}{dt} - 2\nu\varepsilon \right\|_{t,\infty}}{max\left(\frac{dE_k}{dt}\right)^\star},$$

$$\|a\|_{t,\infty} = \max_{\in[0,10]} |a|.$$

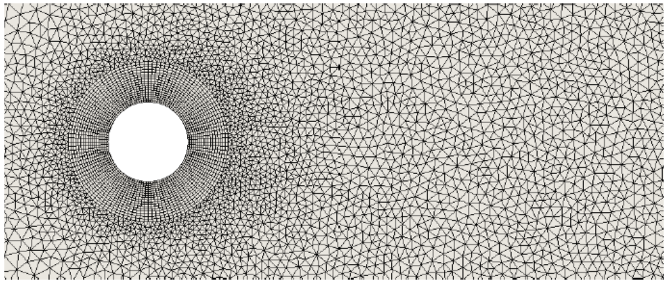
2nd-O upwind

3rd-O MUSCL/CD

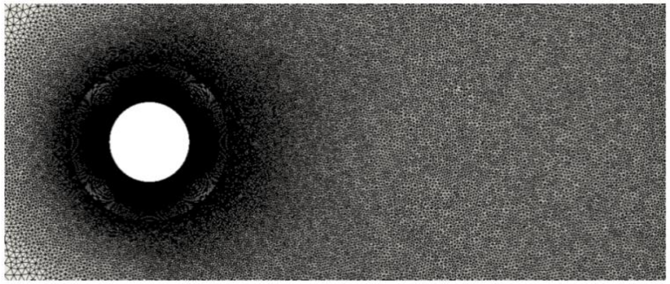
# LES of Circular Cylinder at Re=3,900

- Ma = 0.2      x [-9D, 25D], y [-9D, 9D], z [0,πD]      Coupled implicit solver
- periodic in z, free-steam in, pressure outlet for other outer boundaries
- Initialised with the Grid-sequencing (solves inviscid flow on coarse-grid levels based on the linear-system coefficients, from coarsest mesh to fine mesh)
- WALE subgrid-scale model (none for PyFR)
- Time statistics collected over 100-1100 $t_c$
- dt = 8.5e-2 tc (3O)
- dt = 5.0e-3 tc (2O)
- dt = 2.4e-4 tc (PyFR)
- Ran on 256 cores for 2-3 weeks

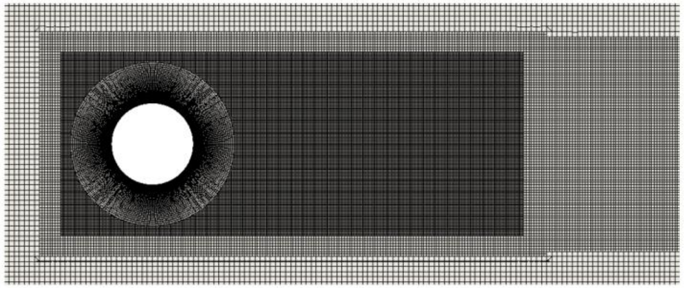
Prismatic+ Tetra. HO mesh (307K, **13.9M** on P4 elements)



Prismatic+ Tetra. mesh (**13.2M**)



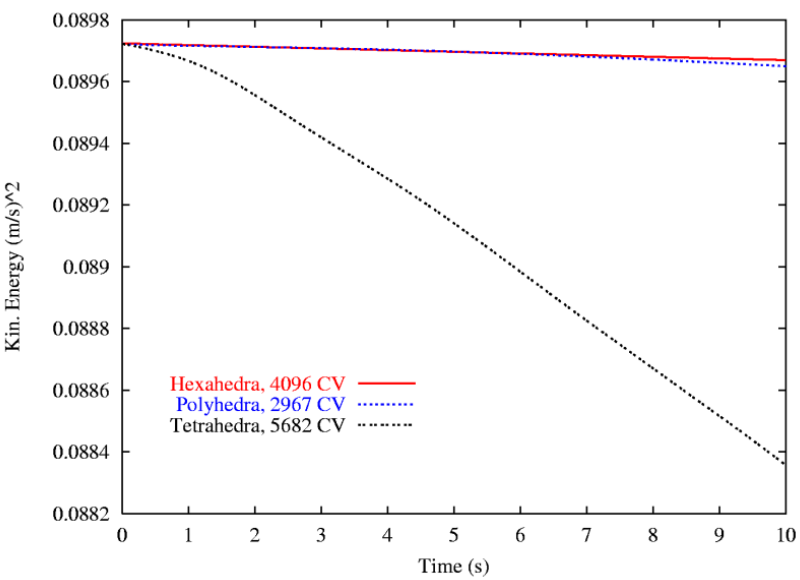
Prismatic+ hexa. mesh (**13.5M**)



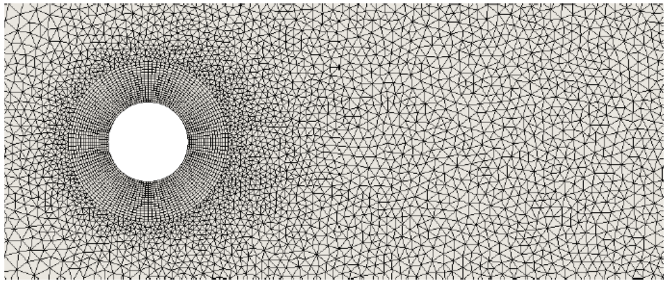
# LES of Circular Cylinder at Re=3,900

- Ma = 0.2 x [-9D, 25D], y [-9D, 9D], z [0,πD] Coupled implicit solver
- periodic in z, free-steam in, pressure outlet for other outer boundaries
- Initialised with the Grid-sequencing (solves inviscid flow on coarse-grid levels based on the linear-system coefficients, from coarsest mesh to fine mesh)

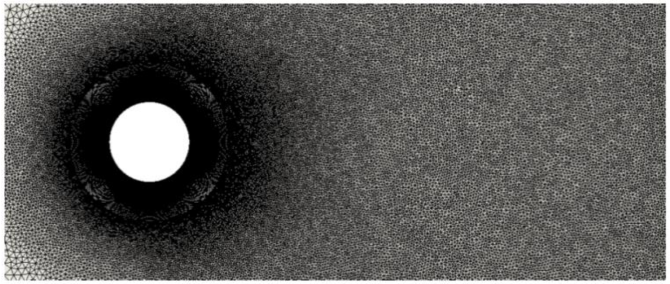
Tetrahedral meshes much more diffusive (see below conservation of kinetic energy for 2D Taylor Green)



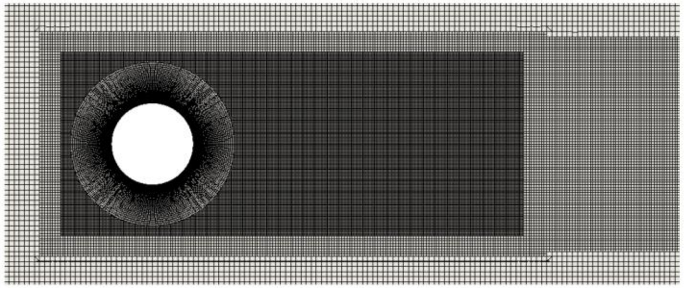

Prismatic+ Tetra. HO mesh (307K, **13.9M** elements)



Prismatic+ Tetra. mesh (**13.2M**)



Prismatic+ hexa. mesh (**13.5M**)

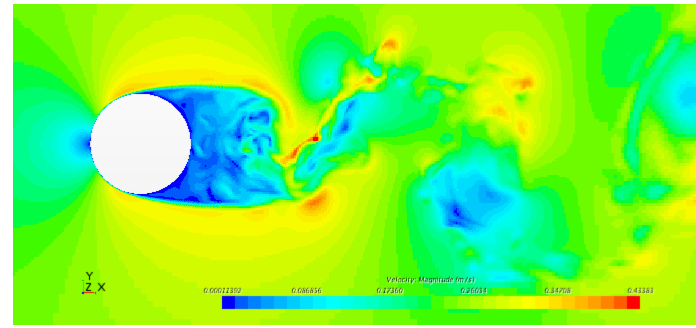


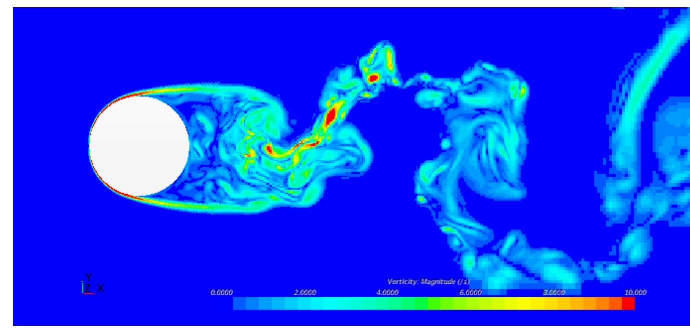
# LES of Circular Cylinder at Re=3,900

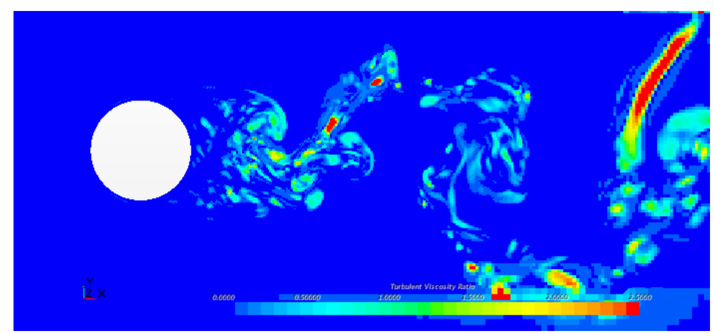
| | $L_z/D$ | $t_c U_\infty/D$ | $f_{vs}$ | $C_d$ |
|---|---|---|---|---|
| Present LES (3O MUSCL/CD) | $\pi$ | 1000 | 0.215 | 1.053* |
| DNS (Lehmkuhl et al. 2013) | $\pi$ | 3900 | 0.211 | 1.015 |
| DNS (Ma et al. 2000) | $2\pi$ | 480 | 0.203 | 0.96 |
| DNS (Tremblay 2002) | $\pi$ | 240 | 0.22 | 1.03 |
| LES (Kravchenko & Moin, 2000) | $\pi$ | 35 | 0.21 | 1.04 |
| Exp. (Norberg 1988) (Re=3,000) | 67 | -- | 0.22 | 0.98 |

* $C_d$ only over 280 $t_c$

Velocity magnitude, vorticity magnitude & SGS viscosity ratio



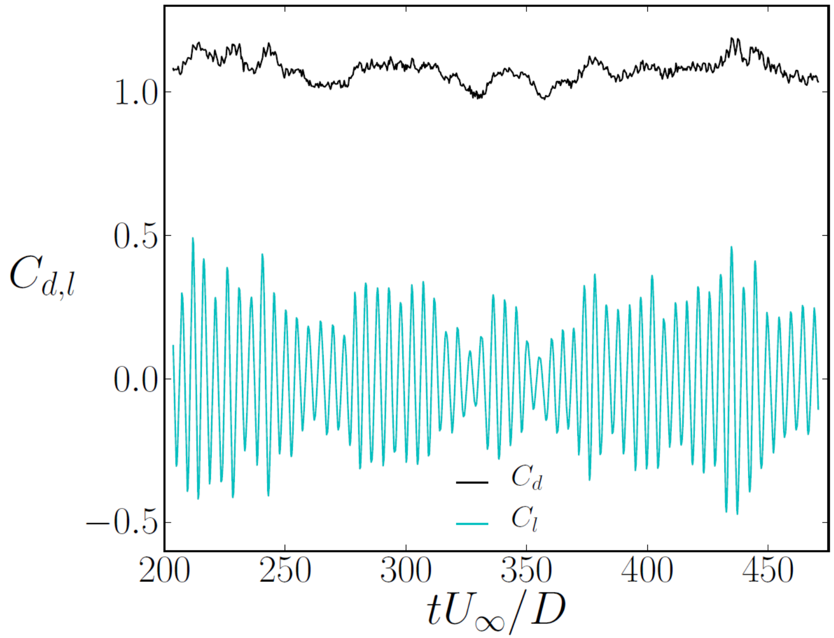


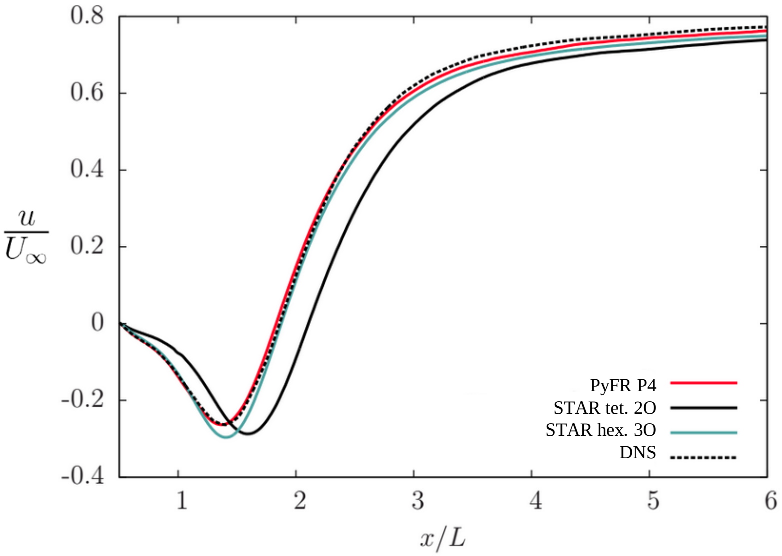


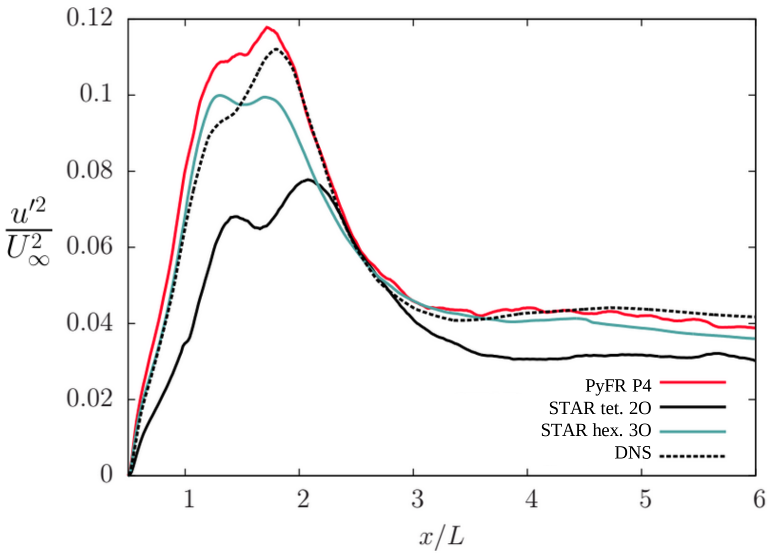
# LES of Circular Cylinder at Re=3,900

Time evolution of lift and drag showing low-frequency unsteadiness/undulations

Time and Spanwise averaged streamwise velocity and fluctuations along streamwise direction from the cylinder surface

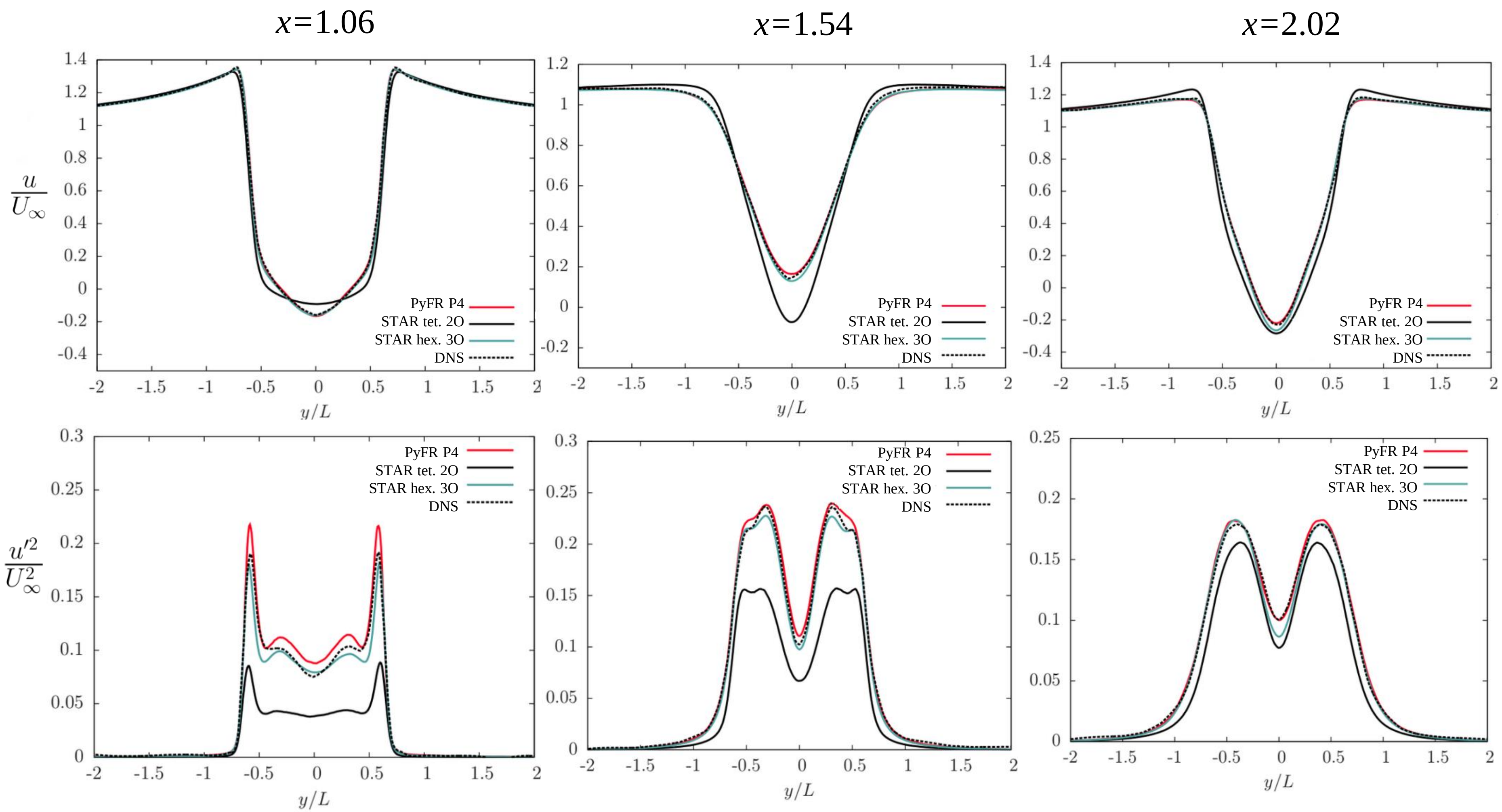


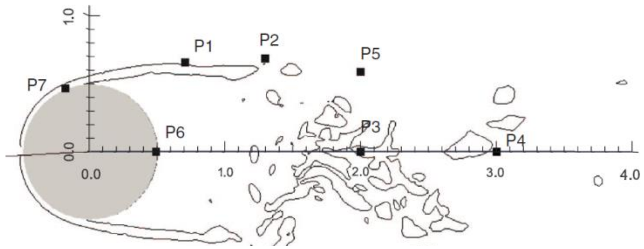
# LES of Circular Cylinder at Re=3,900

- Time and Spanwise averaged streamwise velocity and fluctuations along streamwise slices

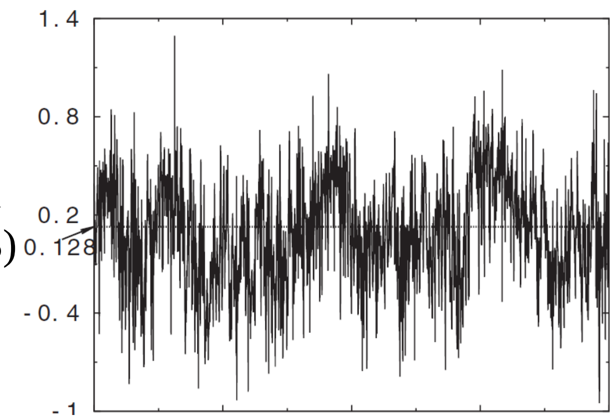


# LES of Circular Cylinder at Re=3,900

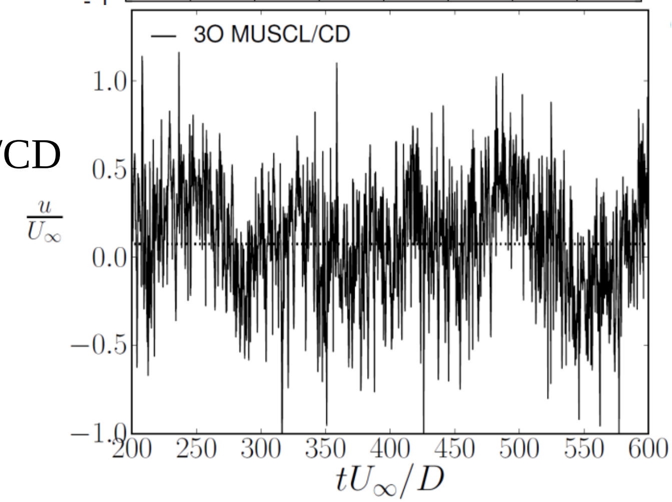
Point velocity fluctuations and spectra




P3 DNS (Lehmkuhl et al., 2013)

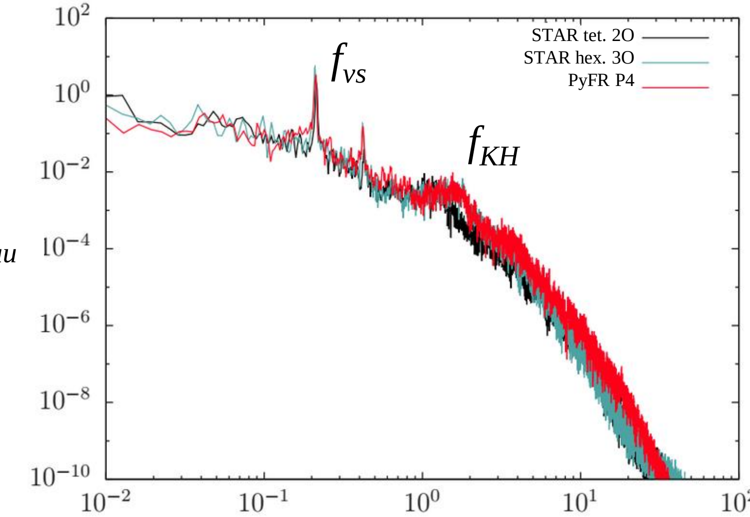



P3 3O MUSCL/CD



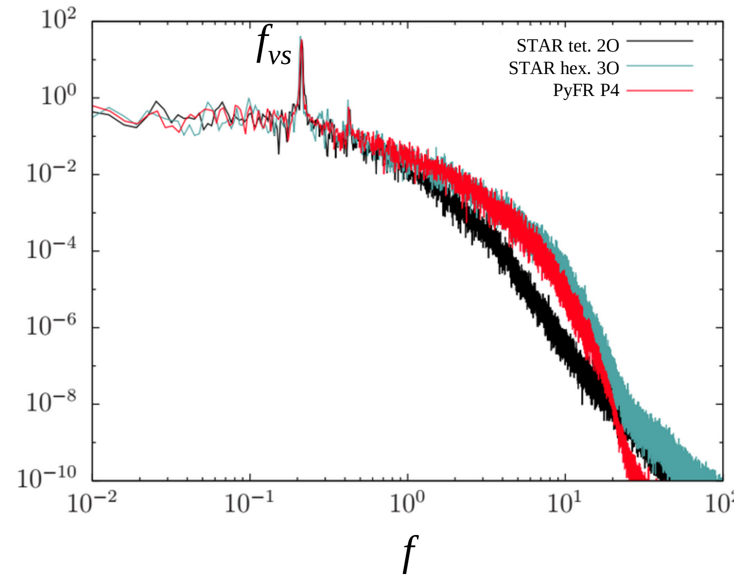

P1

$E_{uu}$

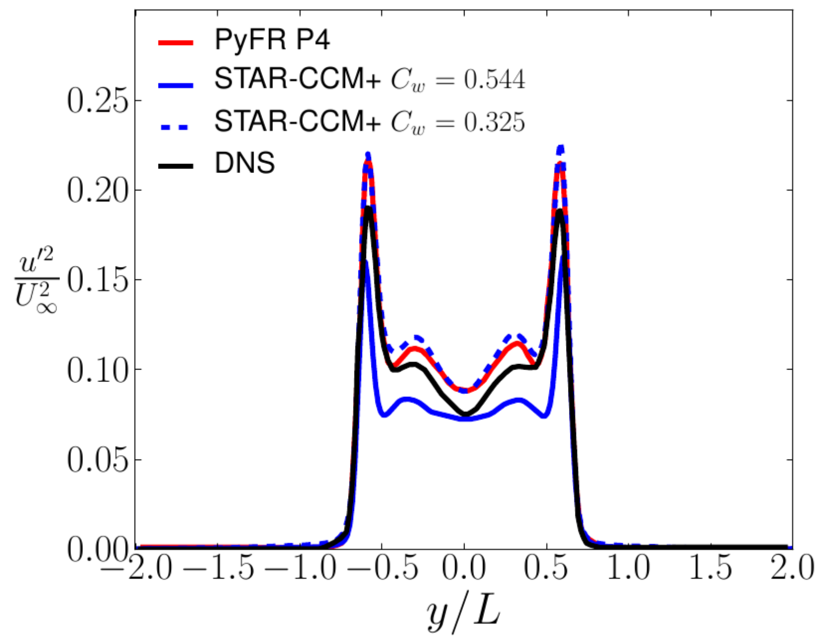



P5

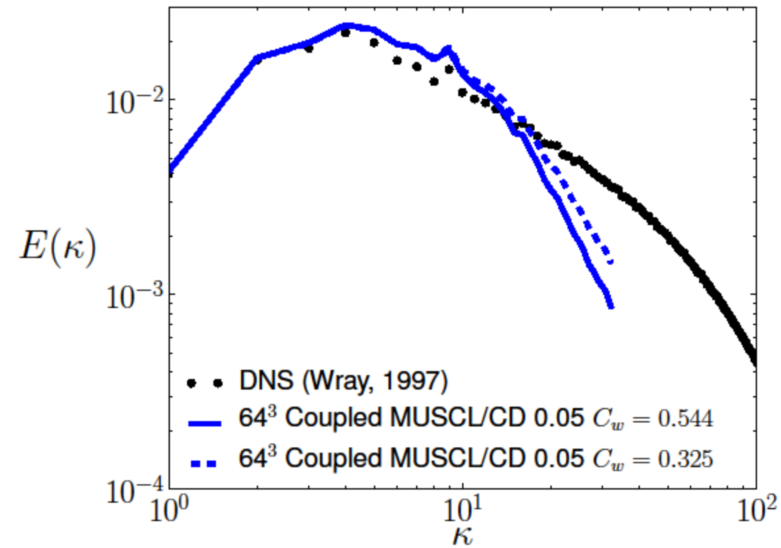
$E_{uu}$



# LES of Circular Cylinder at Re=3,900




Effect of decreasing Cw to widely accepted value




Computational resource:

| | Hardware | CUDA C Cores/cores x nodes | Price (£) | GPU/ CPU hours | Resource Utilization (£×sec) |
|---|---|---|---|---|---|
| PyFR P4 | Nvidia K20c GPU | 2496x3x12 | ~2000 | $4.24\times10^4$ | $3.05\times10^{11}$ |
| 2O upwind | Xeon X5650 CPU | 6 x 5 | ~700 | $6.96\times10^4$ | $1.76\times10^{11}$ |
| 3O MUSCL/CD | Xeon X5650 CPU | 6 x 5 | ~700 | $1.39\times10^5$ | $3.51\times10^{11}$ |

# Conclusions

- 3rd Order hybrid MUSCL/CD scheme was validated on fundamental test cases

- Formal order accuracy still to be proven on fundamental cases

- 3rd Order scheme compares well against higher-order codes for more complex high-fidelity simulations (both in terms of accuracy and compute resource)

- Work still needed to single out contibution to mesh and convection scheme

- Future work on proving formal convergence rate (using other HO workshop cases)

- Acknowledgements:

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