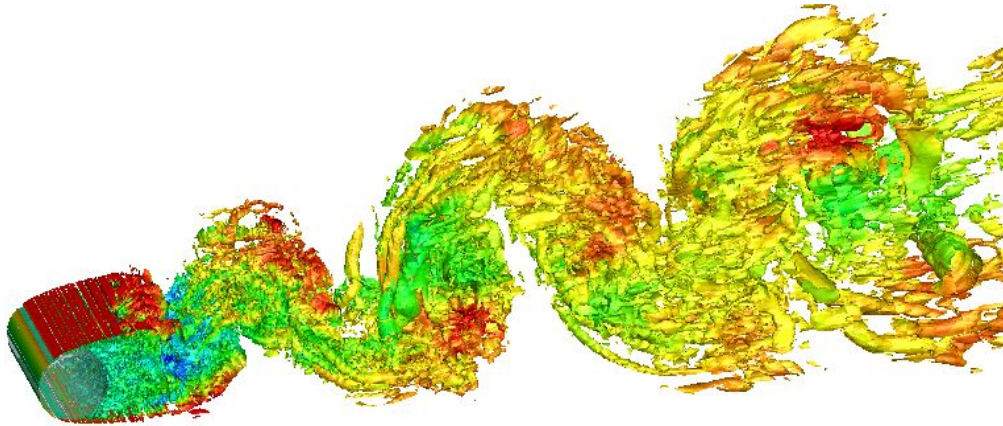


## AS1 - LES of the transitional flow around an infinite cylinder at $Re=3900$



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### General description

This test case is aimed at characterizing the accuracy and efficiency of high-order solvers for the prediction of complex unsteady multi-scale problems under low Reynolds number conditions. An infinitely smooth initial condition with a bias in the span-wise and vertical direction is employed, and the unsteady solution at a time ( $t_1$ ) before any chaos or turbulence starts will be used to assess the spatial and time accuracy of the flow solver. The bias is designed to ensure that the non-symmetric flow is not a consequence of round-off or truncation errors. After the flow reaches a statistically periodic state at  $t_2$ , averaged quantities and Reynolds stresses are computed until  $t_3$  for comparison purposes. Since  $t_1$  is expected to be relatively small, hp-refinements or adaptations will be employed to obtain accurate  $C_l$  and  $C_d$  (error < 0.01 count) values at  $t_1$  to assess solution accuracy.

The following non-dimensional times are specified:

$$t_1 = 1, t_2 = 100, t_3 = 800$$

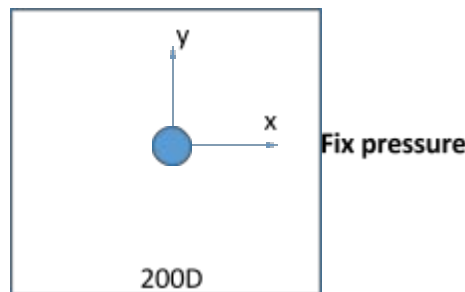
The  $C_l$  and  $C_d$  values obtained by several groups are:  $C_l = 0.070603$  and  $C_d = -0.151505$ , for your reference. All times are non-dimensionalized based on the characteristic time scale:  $t^* = D/U_\infty$ .

## Governing Equations

The governing equations are the full 3D compressible Navier-Stokes equations with a constant ratio of specific heats of 1.4, a constant viscosity, and a Prandtl number of 0.71 with inflow Mach of 0.3. Given the low value of Reynolds number being considered, emphasis is placed on ILES approaches; however, methodologies which incorporate dynamic sub-grid-scale (SGS) models are also of interest.

## Geometry

The diameter of the cylinder is  $D = 1$ . In order to minimize computational cost, the span-wise dimension is  $L = 2xD$ , and the far-field is a square  $100D$  away from the center of the cylinder, as shown in the following figure. The Reynolds number based on the diameter is 3,900.



## Flow Conditions

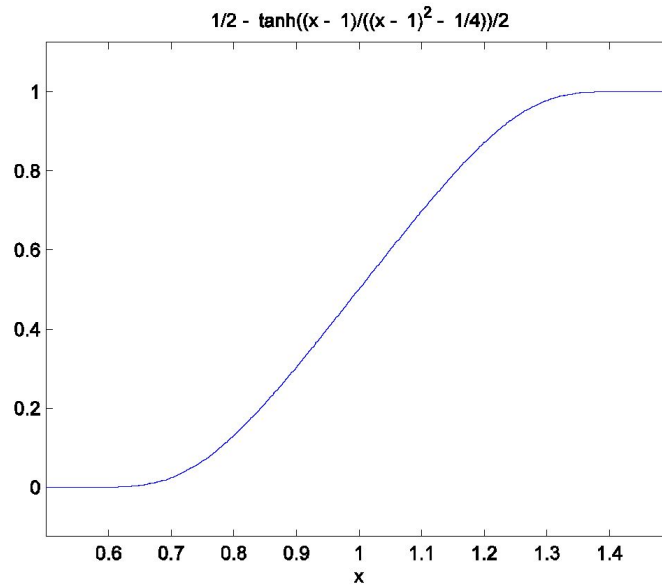
The upstream Mach number is  $M=0.3$ , the Reynolds number based on  $D$  is 3,900, and the angle of attack is  $0^\circ$ .

The initial condition is infinitely smooth. The free stream is blended smoothly with a no slip condition on the cylinder surface. In addition, the initial flow has a bias (non-symmetric) in the span-wise and vertical direction ( $y$ ).

Let  $(r, \theta)$  be the polar coordinates on the  $x$ - $y$  plane. The cylinder surface is  $r = 0.5$ , and the span goes from 0 to  $L$ . Define a smooth function between 0 and 1.

$$S(r, r_m, r_a) = 0.5 \left( 1 + \tanh \left( \frac{2r_a(r - r_m)}{r_a^2 - (r - r_m)^2} \right) \right), \quad r_m - r_a \leq r \leq r_m + r_a$$

This function goes from zero to 1 between  $r = r_m - r_a$  and  $r_m + r_a$ , with zero derivatives of any order at both end points as shown in the following figure.



The free stream is  $p = p_\infty, T = T_\infty, u = U_\infty, v = 0, w = 0$ . The pressure and temperature are constant in the entire computational domain.

If  $r < 1.5$

$$u = U_\infty \cdot S(r, 1, 0.5)$$

If  $0.5 < r \leq 1$

$$w = \varepsilon \cdot U_\infty \cdot S(r, 0.75, 0.25) \cdot \sin^2(\pi z / L)$$

Else

$$w = \varepsilon \cdot U_\infty \cdot (1 - S(r, 1.25, 0.25)) \cdot \sin^2(\pi z / L)$$

End if

If  $0 < \theta < \pi$

If  $0.5 < r \leq 1$

$$v = \varepsilon \cdot U_{\infty} \cdot S(r, 0.75, 0.25) \cdot S\left(\theta, \frac{\pi}{2}, \frac{\pi}{2}\right)$$

Else

$$v = \varepsilon \cdot U_{\infty} \cdot (1 - S(r, 1.25, 0.25)) \cdot S\left(\theta, \frac{\pi}{2}, \frac{\pi}{2}\right)$$

End if

Else

If  $0.5 < r \leq 1$

$$v = \varepsilon \cdot U_{\infty} \cdot S(r, 0.75, 0.25) \cdot \left(1 - S\left(\theta, \frac{3\pi}{2}, \frac{\pi}{2}\right)\right)$$

Else

$$v = \varepsilon \cdot U_{\infty} \cdot (1 - S(r, 1.25, 0.25)) \cdot \left(1 - S\left(\theta, \frac{3\pi}{2}, \frac{\pi}{2}\right)\right)$$

Endif

Endif

Endif

The parameter  $\varepsilon$  is 0.1.

The freestream flow conditions are :  $p_{\infty} = 1, T_{\infty} = 1/1.4, \rho_{\infty} = 1.4, U_{\infty} = 0.3$

## Boundary Conditions

Exit plane: fix pressure

Cylinder surface: no slip adiabatic wall

Span-wise: periodicity

Rest: characteristic inflow and outflow.

## Mandatory computations and results

1. Run the simulation until  $t_1$ , and find the  $C_l$  and  $C_d$ . Perform time and mesh refinement studies to find accurate values with 0.01 count accuracy, and use the values to compute  $C_l$  and  $C_d$  errors. For the short term simulation, you may be able to use a much finer resolution than for the long term simulation. For the long term simulation, you can choose any mesh resolution and order.
2. Plot  $C_l$  and  $C_d$  histories vs. non-dimensional time
3. Compute the mean flow and Reynolds stresses with spanwise averaging between  $t_2$  and  $t_3$ .

4. Compute the mean u-velocity and Reynolds stresses ( $u'u'$ ,  $u'v'$ ,  $v'v'$ ) at prescribed wake stations between  $-3 < y < 3$ , at  $x/D = 0.58, 1.54, 6, 10$ . Again average the results in the spanwise direction.
5. Compute the mean pressure and surface skin friction coefficients with spanwise averaging on the cylinder surface between  $t_2$  and  $t_3$ .
6. Frequency spectra for total velocity, pressure coefficient, and turbulent kinetic energy at selected wake points  $(0.58, 0, L/2)$ ,  $(1.54, 0, L/2)$ ,  $(5, 0, L/2)$ ,  $(10, 0, L/2)$
7. Provide computational resources in terms of dof and work units. In addition, provide the  $C_l$  and  $C_d$  errors at  $t_1$ , which serve as an error indicator.

Note: For all outputs, use non-dimensional data. For example, scale Reynolds stresses by the square of the freestream velocity.